RECOGNISING ACHIEVEMENT

## ADVANCED GCE UNIT <br> MATHEMATICS

## Mechanics 2

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72 .


## ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are reminded of the need for clear presentation in your answers.

1 A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of $35^{\circ}$ with the horizontal and the tension in the rope is 40 N . Calculate the work done in moving the sack 100 m .

2 Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of elevation of $27^{\circ}$.

3 A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100 \mathrm{~m} \mathrm{~s}^{-1}$ to $150 \mathrm{~m} \mathrm{~s}^{-1}$ in a time $t$ seconds.
(i) Calculate the value of $t$.
(ii) Calculate the acceleration of the rocket at the instant when its speed is $120 \mathrm{~m} \mathrm{~s}^{-1}$.

4 A ball is projected from a point $O$ on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7 \mathrm{~m} \mathrm{~s}^{-1}$ and $21 \mathrm{~m} \mathrm{~s}^{-1}$ respectively. At time $t$ seconds after projection the ball is at the point $(x, y)$ referred to horizontal and vertically upward axes through $O$. Air resistance may be neglected.
(i) Express $x$ and $y$ in terms of $t$, and hence show that $y=3 x-\frac{1}{10} x^{2}$.

The ball hits the sea at a point which is 25 m below the level of $O$.
(ii) Find the horizontal distance between the cliff and the point where the ball hits the sea.

5 A cyclist and her bicycle have a combined mass of 70 kg . The cyclist ascends a straight hill $A B$ of constant slope, starting from rest at $A$ and reaching a speed of $4 \mathrm{~m} \mathrm{~s}^{-1}$ at $B$. The level of $B$ is 6 m above the level of $A$. For the cyclist's motion from $A$ to $B$, find
(i) the increase in kinetic energy,
(ii) the increase in gravitational potential energy.

During the ascent the resistance to motion is constant and has magnitude 60 N . The work done by the cyclist in moving from $A$ to $B$ is 8000 J .
(iii) Calculate the distance $A B$.

6


A particle $P$ of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point $A$ and the other end of the shorter string is attached to a fixed point $B$, which is vertically below $A$. $A P$ makes an angle of $30^{\circ}$ with the vertical and is 0.4 m long. $P B$ makes an angle of $60^{\circ}$ with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string $A P$ is 5 N .

Calculate
(i) the tension in the string $P B$,
(ii) the angular speed of $P$,
(iii) the kinetic energy of $P$.

7 Two small spheres $A$ and $B$, with masses 0.3 kg and $m \mathrm{~kg}$ respectively, lie at rest on a smooth horizontal surface. $A$ is projected directly towards $B$ with speed $6 \mathrm{~m} \mathrm{~s}^{-1}$ and hits $B$. The direction of motion of $A$ is reversed in the collision. The speeds of $A$ and $B$ after the collision are $1 \mathrm{~m} \mathrm{~s}^{-1}$ and $3 \mathrm{~m} \mathrm{~s}^{-1}$ respectively. The coefficient of restitution between $A$ and $B$ is $e$.
(i) Show that $m=0.7$.
(ii) Find $e$.
$B$ continues to move at $3 \mathrm{~m} \mathrm{~s}^{-1}$ and strikes a vertical wall at right angles. The coefficient of restitution between $B$ and the wall is $f$.
(iii) Find the range of values of $f$ for which there will be a second collision between $A$ and $B$.
(iv) Find, in terms of $f$, the magnitude of the impulse that the wall exerts on $B$.
(v) Given that $f=\frac{3}{4}$, calculate the final speeds of $A$ and $B$, correct to 1 decimal place.
[Question 8 is printed overleaf.]


Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N . The cylinder has height $h \mathrm{~m}$. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point $O$ (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from $O$.
(i) Show that $h=1.2$.


Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of $30^{\circ}$ with the horizontal (see Fig. 2). The tension in the string is $T \mathrm{~N}$ and the frictional force acting on the object is $F \mathrm{~N}$.
(ii) By taking moments about $O$, express $F$ in terms of $T$.
(iii) Find another equation connecting $T$ and $F$. Hence calculate the tension and the frictional force.

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