OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4737/01**

MATHEMATICS

Decision Mathematics 2

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
Insert for Questions 3, 4 and 5
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- There is an **insert** for use in Questions **3**, **4** and **5**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **7** printed pages, **1** blank page and an insert.

1 Arnie (*A*), Brigitte (*B*), Charles (*C*), Diana (*D*), Edward (*E*) and Faye (*F*) are moving into a home for retired Hollywood stars. They all still expect to be treated as stars and each has particular requirements.

Arnie wants a room that can be seen from the road, but does not want a ground floor room; Brigitte wants a room that looks out onto the garden; Charles wants a ground floor room; Diana wants a room with a balcony; Edward wants a second floor room; Faye wants a room, with a balcony, that can be seen from the road.

The table below shows the features of each of the six rooms available.

Room	Floor	Can be seen from road	Looks out onto garden	Has balcony
1	Ground	✓		
2	Ground		✓	
3	First		✓	✓
4	First	✓		✓
5	Second		✓	✓
6	Second	✓		

(i) Draw a bipartite graph to show the possible pairings between the stars (*A*, *B*, *C*, *D*, *E* and *F*) and the rooms (1, 2, 3, 4, 5 and 6). [2]

Originally Arnie was given room 4, Brigitte was given room 3, Charles was given room 2, Diana was given room 5, Edward was given room 6 and Faye was given room 1.

(ii) Identify the star that has not been given a room that satisfies their requirements. Draw a second bipartite graph to show the incomplete matching that results when this star is not given a room. [2]

(iii) Construct the shortest possible alternating path, starting from the star without a room and ending at the room that was not used, and hence find a complete matching between the stars and the rooms. Write a list showing which star should be given which room. [2]

When the stars view the rooms they decide that some are much nicer than others. Each star gives each room a value from 1 to 6, where 1 is the room they would most like and 6 is the room they would least like. The results are shown below.

	Room					
	1	2	3	4	5	6
Arnie (*A*)	3	6	4	1	5	2
Brigitte (*B*)	5	3	2	4	1	6
Charles (*C*)	2	1	3	4	5	6
Diana (*D*)	5	4	1	3	2	6
Edward (*E*)	5	6	4	3	2	1
Faye (*F*)	5	6	4	1	3	2

(iv) Apply the Hungarian algorithm to this table, reducing rows first, to find a minimum 'cost' allocation between the stars and the rooms. Write a list showing which star should be given which room according to this allocation. Write down the name of any star whose original requirements are not satisfied. [8]

2 As part of a team-building exercise the reprographics technicians (Team R) and the computer network support staff (Team C) take part in a paintballing game. The game ends when a total of 10 'hits' have been scored.

Each team has to choose a player to be its captain. The number of 'hits' expected by Team R for each pair of captains is shown below.

		Team C		
		Liam	Mike	Nicola
Team R	Philip	4	5	6
	Sanjiv	3	2	4
	Tina	6	5	3

The teams are each trying to maximise their number of hits.

(i) If Team R chooses Philip and Team C chooses Liam, how many hits will Team C expect? [1]

(ii) Explain why subtracting 5 from each of the values in the table will convert the game into a zero-sum game. [1]

(iii) Find the play-safe strategies for the zero-sum game. Explain how you know that the game is not stable. State which choice of captain is best for Team C if they know that Team R will play safe. [5]

(iv) Explain carefully why increasing the expected number of hits for Team R when Philip and Liam are chosen as the captains will not change the play-safe strategies. [2]

(v) Explain why Team R should never choose Sanjiv as the captain. [1]

Team R chooses its captain by using random numbers to choose between Philip and Tina, where the probability of choosing Philip is p and the probability of choosing Tina is $1 - p$.

(vi) Show that the expected number of hits for Team R when Team C choose Liam is given by $6 - 2p$ and find the corresponding expressions when Mike is chosen and when Nicola is chosen. [3]

(vii) Use a graphical method to find the optimal value of p for Team R and how many hits Team R can expect when this value is used. [4]

[Turn over

3 Answer this question on the insert provided.

The table shows a partially completed dynamic programming tabulation for solving a minimax problem.

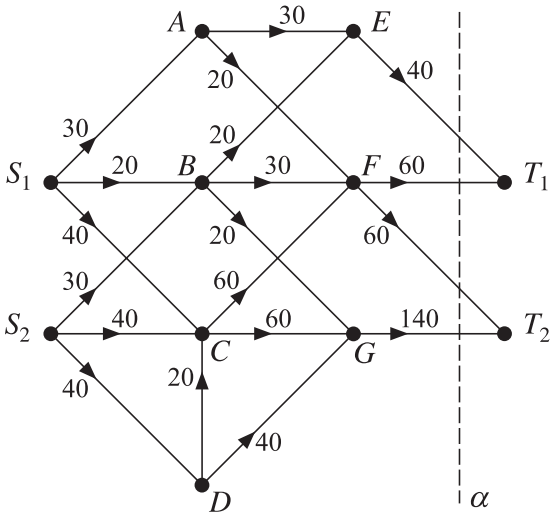
Stage	State	Action	Working	Minimax
1	0	0	1	
	1	0	3	
	2	0	2	
2	0	0	(4,	
		1	(2,	
	1	1	(3,	
		2	(5,	
	2	0	(2,	
		2	(4,	
3	0	0	(5,	
		1	(3,	
		2	(1,	

(i) Complete the last two columns of the table in the insert. [6]

(ii) State the minimax value and write down the minimax route. [3]

(iii) Draw the network represented by the table. [3]

4 Answer this question on the insert provided.



The diagram represents a system of pipes through which fluid can flow from two sources, S_1 and S_2, to two sinks, T_1 and T_2. It also shows a cut α. The weights on the arcs show the capacities of the pipes in gallons per hour.

(i) Add a supersource and a supersink to the network in the insert. Give appropriate weightings and directions to the connecting arcs. [2]

(ii) Explain why the arcs *AE* and *BE* cannot both be full to capacity. [2]

(iii) Calculate the capacity of the cut α. [1]

(iv) Calculate the capacity of the cut X = $\{S_1, S_2, B, D\}$, Y = $\{A, C, E, F, G, T_1, T_2\}$. [2]

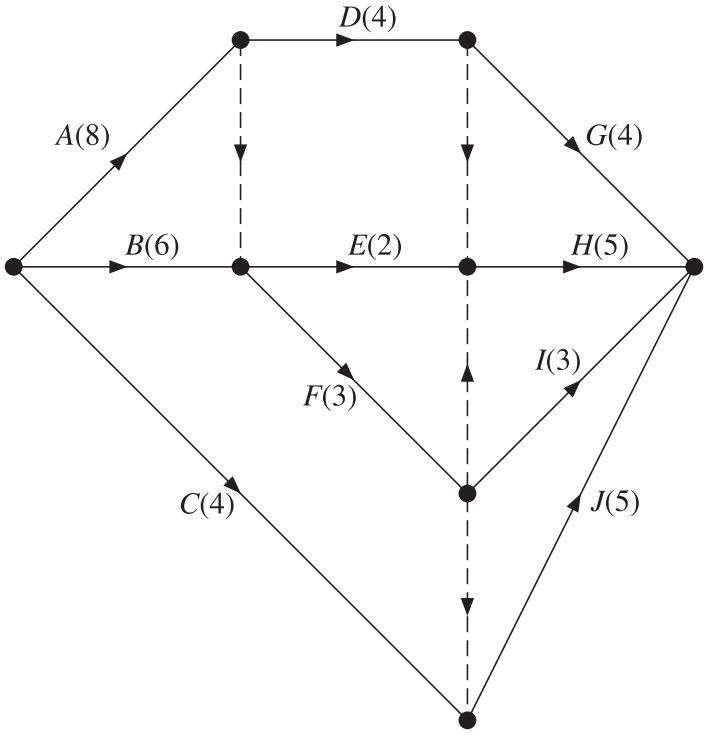
(v) On the diagram in the insert, show a flow through the network of 200 gallons per hour. Show that this flow is maximal by finding a cut of 200 gallons per hour. [3]

(vi) Vertex *C* now has a vertex restriction applied that means that no more than 20 gallons per hour can flow through it. Amend the diagram in the insert to show this restriction. Find the value of the maximum flow with the restriction. [4]

[Turn over

5 Answer this question on the insert provided.

The diagram shows an activity network for a project. The figures in brackets show the durations of the activities in days.



(i) Complete the table in the insert to show the precedences for the activities. [3]

(ii) Use the boxes on the diagram in the insert to carry out a forward pass and a backward pass. Find the minimum project duration and list the critical activities. [6]

The number of people required for each activity is shown in the table below. The workers are all equally skilled at all of the activities.

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Number of workers	4	1	2	2	3	2	3	3	1	2

(iii) On graph paper, draw a resource histogram for the project with each activity starting at its earliest possible time. [2]

(iv) Describe how the project can be completed in 21 days using just six workers. [4]

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Time: 1 hour 30 minutes

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

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- Write your name, centre number and candidate number in the boxes above.
- This insert should be used to answer Questions **3**, **4** and **5**.
- Write your answers to Questions **3**, **4** and **5** in the spaces provided in this insert, and attach it to your answer booklet.

This document consists of **4** printed pages.

[Turn over

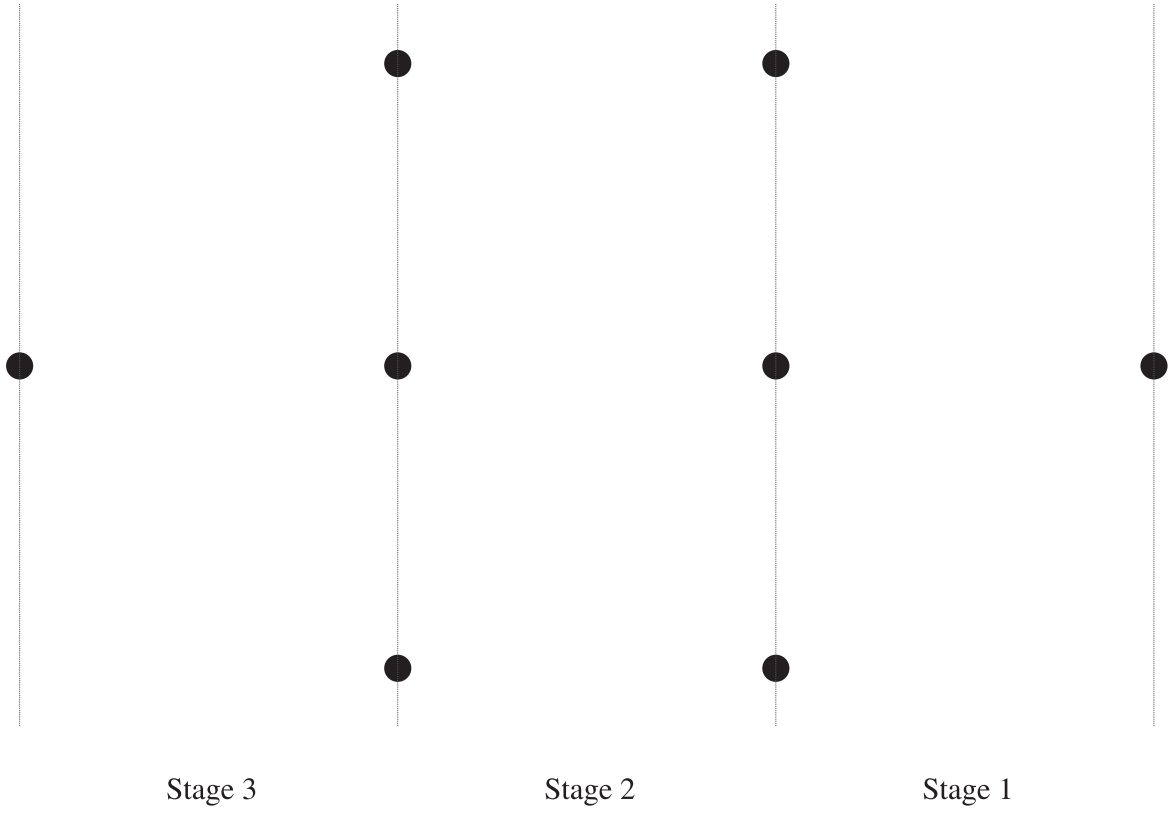
3 (i)

Stage	State	Action	Working	Minimax
1	0	0	1	
	1	0	3	
	2	0	2	
2	0	0	(4,	
		1	(2,	
	1	1	(3,	
		2	(5,	
	2	0	(2,	
		2	(4,	
3	0	0	(5,	
		1	(3,	
		2	(1,	

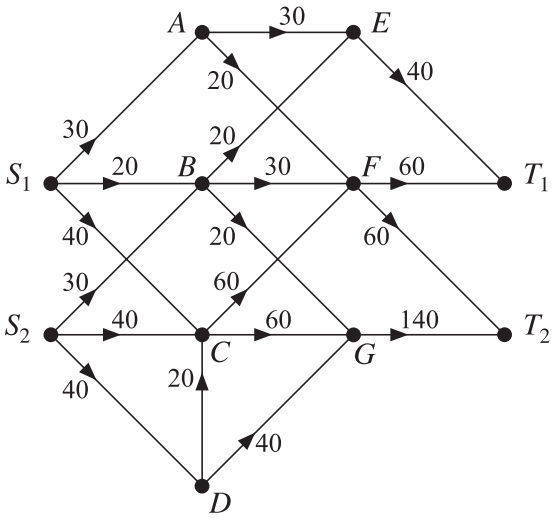
(ii) Minimax value = ...

Minimax route = ...

(iii)



4 **(i)**



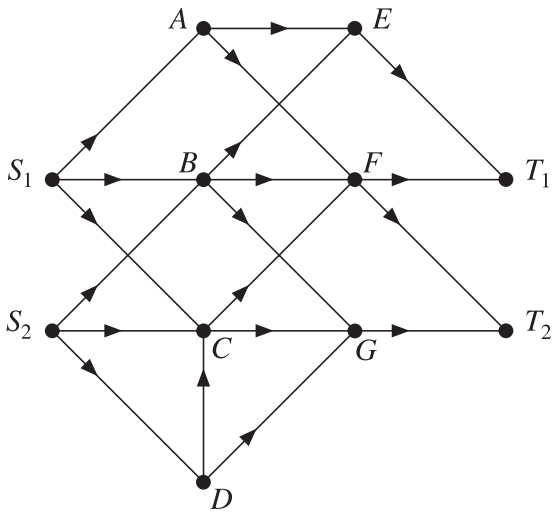
(ii) .....

.....

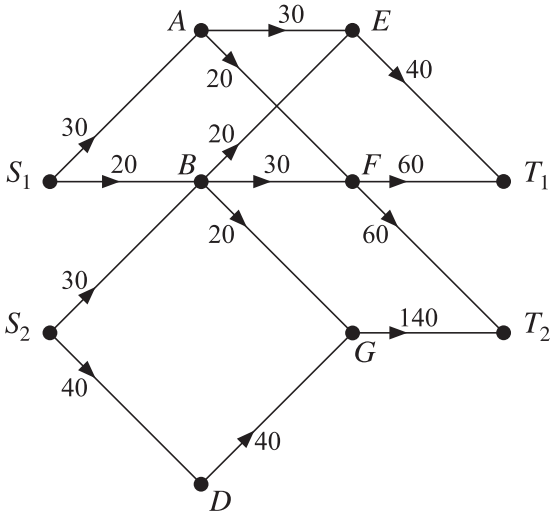
(iii) ..... = ... gallons per hour

(iv) ..... = ... gallons per hour

(v)



(vi)

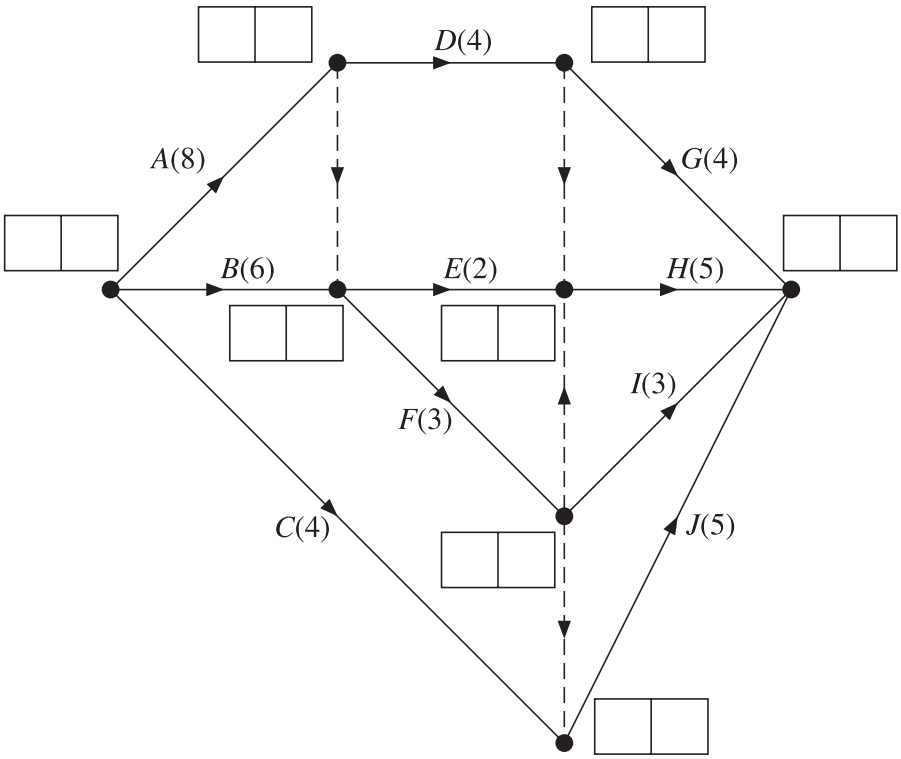


Maximum flow = ... gallons per hour

5 (i)

Activity	Duration (days)	Immediate predecessors
A	8	
B	6	
C	4	
D	4	
E	2	
F	3	
G	4	
H	5	
I	3	
J	5	

(ii)



Minimum project completion time = ... days; critical activities = ...

(iii) To be answered on graph paper.

(iv) .....

.....