

**ADVANCED GCE
MATHEMATICS**

4724/01

Core Mathematics 4

WEDNESDAY 21 MAY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

1 (a) Simplify $\frac{(2x^2 - 7x - 4)(x + 1)}{(3x^2 + x - 2)(x - 4)}$. [2]

(b) Find the quotient and remainder when $x^3 + 2x^2 - 6x - 5$ is divided by $x^2 + 4x + 1$. [4]

2 Find the exact value of $\int_1^e x^4 \ln x \, dx$. [5]

3 The equation of a curve is $x^2y - xy^2 = 2$.

(i) Show that $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$. [3]

(ii) (a) Show that, if $\frac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x -axis. [3]

4 Relative to an origin O , the points A and B have position vectors $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B . [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB . [5]

5 (i) Show that $\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$. [3]

6 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s \begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

7 (i) Show that, if $y = \operatorname{cosec} x$, then $\frac{dy}{dx}$ can be expressed as $-\operatorname{cosec} x \cot x$. [3]

(ii) Solve the differential equation

$$\frac{dx}{dt} = -\sin x \tan x \cot t,$$

given that $x = \frac{1}{6}\pi$ when $t = \frac{1}{2}\pi$. [5]

- 8 (i) Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B . [3]

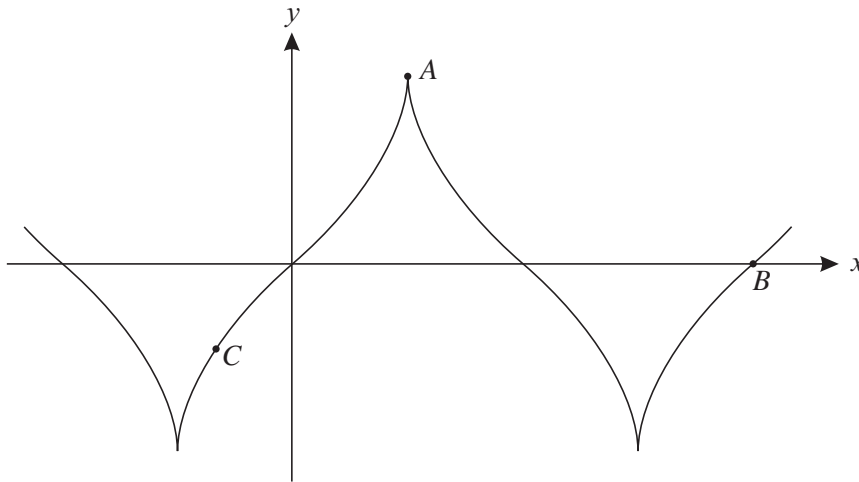
(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x + \sqrt{2x-1}} dx$ to $\int \frac{2t}{(t+1)^2} dt$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x + \sqrt{2x-1}} dx$. [4]

- 9 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 4 \sin \theta,$$

and part of its graph is shown below.



- (i) Find the value of θ at A and the value of θ at B . [3]
- (ii) Show that $\frac{dy}{dx} = \sec \theta$. [5]
- (iii) At the point C on the curve, the gradient is 2. Find the coordinates of C , giving your answer in an exact form. [3]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.