OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Express $\dfrac{1}{(2x+1)(x^2+1)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $\sqrt[3]{1+3x}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[5]**

3 Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2\sin\theta - 3\cos\theta$. **[6]**

4 A curve has parametric equations

$$x = 2\sin\theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter $\theta = \frac{1}{3}\pi$. **[5]**

(ii) Find y in terms of x. **[2]**

5 Solve the equation $\operatorname{cosec}^2\theta = 1 + 2\cot\theta$, for $-180° \leqslant \theta \leqslant 180°$. **[6]**

6 Fig. 6 shows the region enclosed by part of the curve $y = 2x^2$, the straight line $x + y = 3$, and the y-axis. The curve and the straight line meet at P (1, 2).

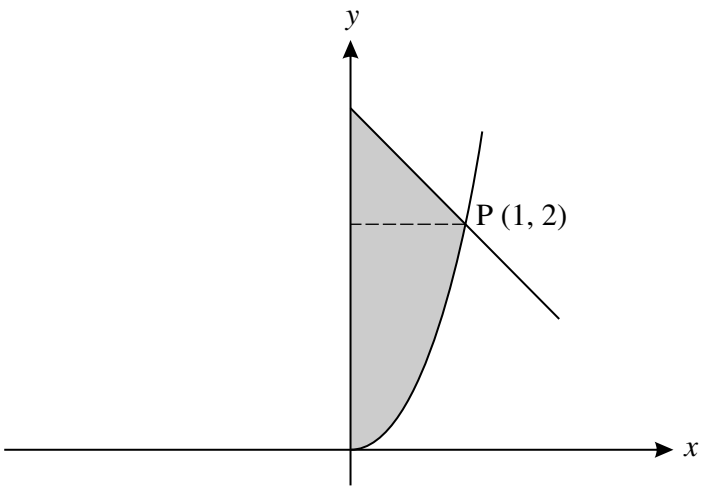

Fig. 6

The shaded region is rotated through 360° about the y-axis. Find, in terms of π, the volume of the solid of revolution formed. **[7]**

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.]

Section B (36 marks)

7 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is k metres.

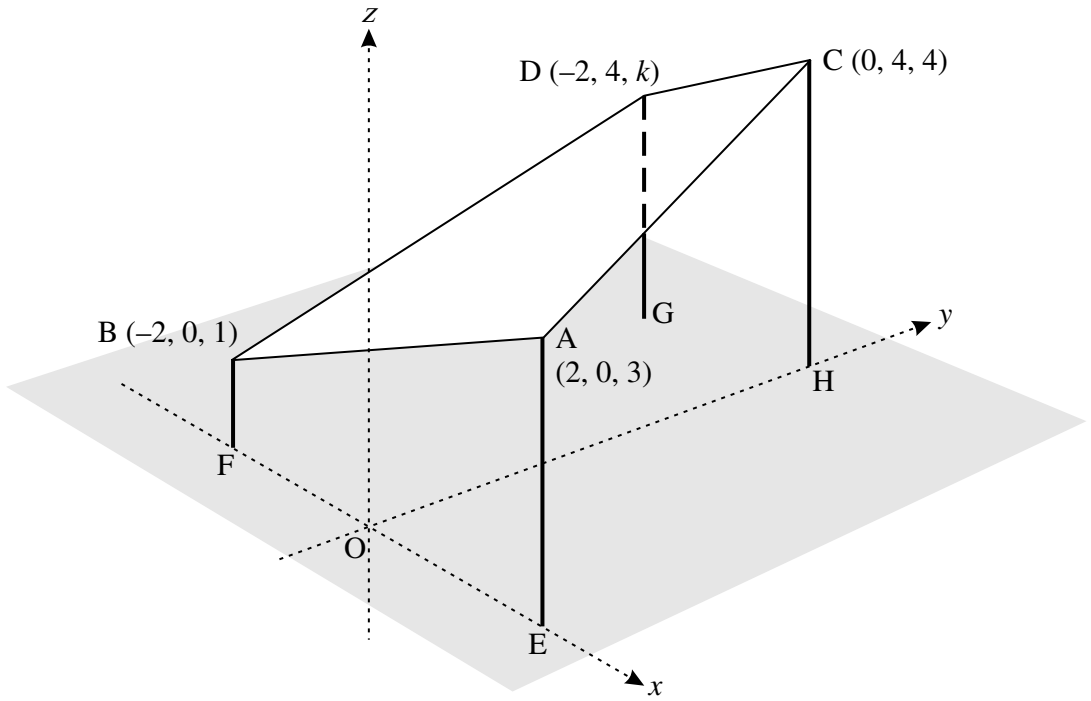

Fig. 7

(i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence calculate the angle BAC. **[6]**

(ii) Verify that the equation of the plane ABC is $x + y - 2z + d = 0$, where d is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. **[7]**

(iii) Given that A, B, D and C are coplanar, show that $k = 3$.

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. **[5]**

[Question 8 is printed overleaf.]

8 Water is leaking from a container. After t seconds, the depth of water in the container is x cm, and the volume of water is V cm^3, where $V = \frac{1}{3}x^3$. The rate at which water is lost is proportional to x, so that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -kx$, where k is a constant.

(i) Show that $x\dfrac{\mathrm{d}x}{\mathrm{d}t} = -k$. **[3]**

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that $x = \sqrt{100 - 2kt}$. **[4]**

(iii) Given that the container empties after 50 seconds, find k. **[2]**

Once the container is empty, water is poured into it at a constant rate of 1 cm^3 per second. The container continues to lose water as before.

(iv) Show that, t seconds after starting to pour the water in, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1-x}{x^2}$. **[2]**

(v) Show that $\dfrac{1}{1-x} - x - 1 = \dfrac{x^2}{1-x}$.

Hence solve the differential equation in part **(iv)** to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \tfrac{1}{2}x^2 - x.$$ **[6]**

(vi) Show that the depth cannot reach 1 cm. **[1]**