

**ADVANCED GCE
MATHEMATICS**

Further Pure Mathematics 3

4727

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

**Monday 13 June 2011
Morning**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A line l has equation $\frac{x-1}{5} = \frac{y-6}{6} = \frac{z+3}{-7}$ and a plane p has equation $x + 2y - z = 40$.

(i) Find the acute angle between l and p . [4]

(ii) Find the perpendicular distance from the point $(1, 6, -3)$ to p . [2]

2 It is given that $z = e^{i\theta}$, where $0 < \theta < 2\pi$, and $w = \frac{1+z}{1-z}$.

(i) Prove that $w = i \cot \frac{1}{2}\theta$. [3]

(ii) Sketch separate Argand diagrams to show the locus of z and the locus of w . You should show the direction in which each locus is described when θ increases in the interval $0 < \theta < 2\pi$. [3]

3 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} + 4y = 5 \cos 3x.$$

(i) Find the complementary function. [2]

(ii) Hence, or otherwise, find the general solution. [7]

(iii) Find the approximate range of values of y when x is large and positive. [2]

4 A group G , of order 8, is generated by the elements a, b, c . G has the properties

$$a^2 = b^2 = c^2 = e, \quad ab = ba, \quad bc = cb, \quad ca = ac,$$

where e is the identity.

(i) Using these properties and basic group properties as necessary, prove that $abc = cba$. [2]

The operation table for G is shown below.

| | e | a | b | c | bc | ca | ab | abc |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| e | e | a | b | c | bc | ca | ab | abc |
| a | a | e | ab | ca | abc | c | b | bc |
| b | b | ab | e | bc | c | abc | a | ca |
| c | c | ca | bc | e | b | a | abc | ab |
| bc | bc | abc | c | b | e | ab | ca | a |
| ca | ca | c | abc | a | ab | e | bc | b |
| ab | ab | b | a | abc | ca | bc | e | c |
| abc | abc | bc | ca | ab | a | b | c | e |

(ii) List all the subgroups of order 2. [2]

(iii) List five subgroups of order 4. [3]

(iv) Determine whether all the subgroups of G which are of order 4 are isomorphic. [2]

- 5 The substitution $y = u^k$, where k is an integer, is to be used to solve the differential equation

$$x \frac{dy}{dx} + 3y = x^2 y^2 \quad (\text{A})$$

by changing it into an equation (B) in the variables u and x .

- (i) Show that equation (B) may be written in the form

$$\frac{du}{dx} + \frac{3}{kx} u = \frac{1}{k} x u^{k+1}. \quad [4]$$

- (ii) Write down the value of k for which the integrating factor method may be used to solve equation (B). [1]

- (iii) Using this value of k , solve equation (B) and hence find the general solution of equation (A), giving your answer in the form $y = f(x)$. [4]

- 6 (a) The set of polynomials $\{ax + b\}$, where $a, b \in \mathbb{R}$, is denoted by P . Assuming that the associativity property holds, prove that P , under addition, is a group. [4]

- (b) The set of polynomials $\{ax + b\}$, where $a, b \in \{0, 1, 2\}$, is denoted by Q . It is given that Q , under addition modulo 3, is a group, denoted by $(Q, +(\text{mod}3))$.

- (i) State the order of the group. [1]

- (ii) Write down the inverse of the element $2x + 1$. [1]

- (iii) $q(x) = ax + b$ is any element of Q other than the identity. Find the order of $q(x)$ and hence determine whether $(Q, +(\text{mod}3))$ is a cyclic group. [4]

- 7 (In this question, the notation ΔABC denotes the area of the triangle ABC .)

The points P , Q and R have position vectors $p\mathbf{i}$, $q\mathbf{j}$ and $r\mathbf{k}$ respectively, relative to the origin O , where p , q and r are positive. The points O , P , Q and R are joined to form a tetrahedron.

- (i) Draw a sketch of the tetrahedron and write down the values of ΔOPQ , ΔOQR and ΔORP . [3]

- (ii) Use the definition of the vector product to show that $\frac{1}{2} |\overrightarrow{RP} \times \overrightarrow{RQ}| = \Delta PQR$. [1]

- (iii) Show that $(\Delta OPQ)^2 + (\Delta OQR)^2 + (\Delta ORP)^2 = (\Delta PQR)^2$. [6]

- 8 (i) Use de Moivre's theorem to express $\cos 4\theta$ as a polynomial in $\cos \theta$. [4]

- (ii) Hence prove that $\cos 4\theta \cos 2\theta \equiv 16 \cos^6 \theta - 24 \cos^4 \theta + 10 \cos^2 \theta - 1$. [1]

- (iii) Use part (ii) to show that the only roots of the equation $\cos 4\theta \cos 2\theta = 1$ are $\theta = n\pi$, where n is an integer. [3]

- (iv) Show that $\cos 4\theta \cos 2\theta = -1$ only when $\cos \theta = 0$. [3]

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