OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4768:** Statistics 3

Mark Scheme for June 2011

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Q1				
(i)	t test might be used because • population variance is unknown • background population is Normal	E1 E1	Allow "sample is small" as an alternative.	**2**
(ii)	H_0: $\mu = 15.3$ H_1: $\mu < 15.3$ where μ is the mean of Gerry's times.	B1 B1	Both hypotheses. Hypotheses in words only must include "population". Do NOT allow " $\bar{X}$ = ... " or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used.	
	$\bar{x} = 14.987$ $s_{n-1} = 0.4567(5)$	B1	$s_n = 0.4333$ but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\dfrac{14.987 - 15.3}{\dfrac{0.45675}{\sqrt{10}}}$ $= -2.167(0)$.	M1 A1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 15.3 + (c's –1.833) $\times \dfrac{0.45675}{\sqrt{10}}$ (= 15.035) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's –1.833) $\times \dfrac{0.45675}{\sqrt{10}}$ (= 15.252) for comparison with 15.3.) c.a.o. but ft from here in any case if wrong. Use of $\mu - \bar{x}$ scores M1A0, but ft.	
	Refer to t_9. Single-tailed 5% point is –1.833.	M1 A1	No ft from here if wrong. Must be minus 1.833 unless absolute values are being compared. No ft from here if wrong. $P(t < -2.167(0)) = 0.0292$.	
	Significant. Seems that Gerry's times have been reduced on average.	A1 A1	ft only c's test statistic. ft only c's test statistic. Conclusion in context to include "average" o.e.	**9**
(iii)	A 5% significance level means that the probability of rejecting H_0 given that it is true is 0.05. Decreasing the significance level would make it less likely that a true H_0 would be rejected. Evidence for rejecting H_0 would need to be stronger.	E1 E1 E1	Or equivalent. Allow answers that relate to the context of the question.	**3**
(iv)	CI is given by 14.987 ±	M1	ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_9 is OK.	
	2·262	B1		
	$\times \dfrac{0.45675}{\sqrt{10}}$	M1		
	= 14.987 ± 0.3267= (14.66(0), 15.31(3))	A1	c.a.o. Must be expressed as an interval.	**4**
				18

Q2

(i)

No. particles	0	1	2	3	4	5
Obs fr	4	7	10	20	17	
Prob'y	0.0150	0.0630	0.1322	0.1852	0.1944	
Expfr	1.50	6.30	13.22	18.52	19.44	
Contrib to X^2	(4.1667)	(0.0778)	0.7843	0.1183	0.3063	
Combined	11 7.80 1.3128					

	Answer	Mark	Guidance	Total
(i)		M1	Probs correct to 3d.p. or better.	
		M1	$\times$ 100 for expected frequencies.	
		A1	All correct.	
	$X^2 = 1.3128 + 0.7843 + 0.1183 + 0.3063 + 0.1083 + 0.1813 + 0.6676 + 0.4056$	M1	Merge first 2 cells.	
		M1	Calculation of X^2.	
	$= 3.884(5)$	A1	c.a.o. (For ungrouped cells $X^2 = 6.816$.)	
	H_0: The Poisson model fits the data.	B1	Ignore any reference to the parameter.	
	H_1: The Poisson model does not fit the data.	B1	Do not accept "data fit model" oe.	
	Refer to χ^2_6.	M1	Allow correct df (= cells – 2) from wrongly grouped table and ft. Otherwise, no ft if wrong.	
	Upper 10% point is 10.64.	A1	No ft from here if wrong. ($\chi^2_7 = 12.02$) $P(X^2 > 3.884) = 0.6924$.	
	Not significant.	A1	ft only c's test statistic.	
	Evidence suggests that the model fits the data.	A1	ft only c's test statistic. Do not accept "data fit model" oe.	**12**
(ii)	H_0: $m = 15$ H_1: $m > 15$	B1	Both. Accept hypotheses in words.	
	where m is the population median diameter(in μm).	B1	Adequate definition of m to include "population".	
	Given $W_- = 53$ ($\therefore$ $W_+ = 157$)			
	Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 20$.	M1	No ft from here if wrong.	
	Lower 5% point is 60 (or upper is 150 if W_+ used).	A1	i.e. a 1-tail test. No ft from here if wrong.	
	Result is significant.	A1	ft only c's test statistic.	
	Evidence suggests that the median diameter appears to be more than 15 μm.	A1	ft only c's test statistic. Conclusion in context to include "average" o.e.	**6**
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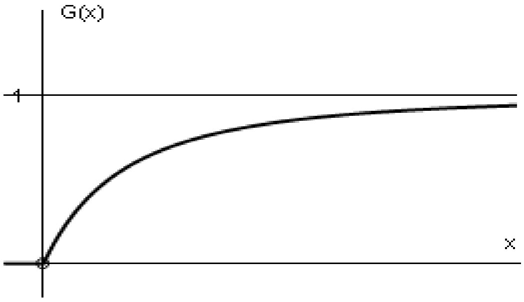
Q3				
(i) (A)		M1 A1 A1	Increasing curve, through (0, 0), in first quadrant only. Asymptotic behaviour. Asymptote labelled; condone absence of axis labels.	**3**
(B)	For the UQ G(u) = 0.75 $\therefore \left(1+\frac{u}{200}\right)^{-2} = \frac{1}{4} \quad \therefore u = 200$ For the LQ G(l) = 0.25 $\therefore \left(1+\frac{l}{200}\right)^{-2} = \frac{3}{4} \quad \therefore l = 200\left(\frac{2}{\sqrt{3}}-1\right) = 30.94...$ $\therefore$IQR = 200 – 30.94 = 169(.06) For an outlier x > UQ +1.5 × IQR = 200 + 1.5 × 169 =453(.58) ≈ 454 (nearest hour)	M1 A1 A1 M1 M1 E1	Use of G(x) for either quartile. c.a.o. c.a.o. UQ – LQ UQ +1.5 × IQR. Answer given; must be obtained genuinely.	**6**
(ii) (A)	$F(x) = \int_0^x \frac{1}{200} e^{\frac{-t}{200}} dt$ $= \left[- e^{\frac{-t}{200}}\right]_0^x = \left(-e^{\frac{-x}{200}}\right) - \left(-e^{\frac{-0}{200}}\right) = 1 - e^{\frac{-x}{200}}$	M1 A1 E1	Correct integral, including limits (which may be implied subsequently). Correctly integrated. Limits used. Answer given; must be shown convincingly. Condone the omission of x < 0 part. Allow use of “+ c” with F(0) = 0.	**3**
(B)	P(X > 50) = 1 – F(50) $= e^{\frac{-50}{200}} = e^{-0.25}$	M1 E1	Use of 1 – F(x) Answer given: must be convincing. (= 0.7788(0))	**2**
(C)	$P(X > 400) = e^{\frac{-400}{200}} = 0.1353(35)$ $P(X > 450) = e^{\frac{-450}{200}} = 0.1053(99)$ $P(X > 450 \mid X > 400) = \frac{P(X > 450)}{P(X > 400)}$ $= \frac{e^{\frac{-450}{200}}}{e^{\frac{-400}{200}}} = e^{\frac{-50}{200}} = e^{-0.25} \; (= 0.7788)$	B1 B1 M1 A1	Accept any form. Accept any form. Conditional probability. Not P(X > 50) × P(X > 400) unless <u>clearly</u> justified. Accept division of decimals, 3dp or better. Accept a.w.r.t. 0.778 or 0.779.	**4**
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Q4	$C \sim \mathrm{N}(10,\ 0.4^2)$, $D \sim \mathrm{N}(35,\ 3.5^2)$ When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.			
(i)	$\mathrm{P}(C < 9.5) = \mathrm{P}\left(Z < \frac{9.5-10}{0.4} = -1.25\right)$ $= 1 - 0{\cdot}8944 = 0.1056$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	$D - S = D - (C_1 + C_2 + C_3 + C_4) \sim \mathrm{N}(-5,$ $\sigma^2 = 3.5^2 + (0.4^2 + 0.4^2 + 0.4^2 + 0.4^2) = 12.89)$ Want P($D > S$) = P($D - S > 0$) $= 1 - \Phi\left(\frac{0-(-5)}{3.59} = 1.39(27)\right)$ $= 1 - 0.9182 = 0.0818$	B1 B1 M1 A1	Mean. Accept +5 for $S - D$. Variance. Accept sd (= 3.590…). Formulation of requirement. Accept S – D < 0. This mark could be awarded in (iii) if not earned here. c.a.o.	4
(iii)	*New* $(D - S) = (D \times 1.3) - (C_1 + ... + C_5) \sim \mathrm{N}(-4.5,$ $\sigma^2 = (3.5^2 \times 1.3^2) + (0.4^2 + ... + 0.4^2) = 21.5025)$ Again want P($D > S$) = P($D - S > 0$) $= 1 - \Phi\left(\frac{0-(-4.5)}{4.637} = 0.9704\right)$ $= 1 - 0.8341 = 0.1659$	B1 M1 A1 A1	Mean. Accept +4.5 for $S - D$. Correct use of $\times 1.3^2$ for variance. c.a.o. Accept sd (= 4.637…) Or S – D < 0. M1 for formulation in (ii) available here. c.a.o.	4
(iv)	CI is given by $9.73 \pm$ 1.96 $\times \frac{0.4}{\sqrt{12}}$ $= 9.73 \pm 0.2263 = (9.50(37),\ 9.95(63))$ Since 10 lies above this interval, it seems that the cheeses are underweight. In repeated sampling, 95% of all confidence intervals constructed in this way will contain the true mean.	M1 B1 M1 A1 E1 E1 E1	1.96 seen. c.a.o. Must be expressed as an interval. Ft c's interval.	7
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