OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4733/01**

MATHEMATICS

Probability & Statistics 2

TUESDAY 15 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 The random variable T is normally distributed with mean μ and standard deviation σ. It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of μ and σ. [6]

2 A village has a population of 600 people. A sample of 12 people is obtained as follows. A list of all 600 people is obtained and a three-digit number, between 001 and 600 inclusive, is allocated to each name in alphabetical order. Twelve three-digit random numbers, between 001 and 600 inclusive, are obtained and the people whose names correspond to those numbers are chosen.

(i) Find the probability that all 12 of the numbers chosen are 500 or less. [3]

(ii) When the selection has been made, it is found that all of the numbers chosen are 500 or less. One of the people in the village says, "The sampling method must have been biased." Comment on this statement. [2]

3 The random variable G has the distribution $Po(\lambda)$. A test is carried out of the null hypothesis $H_0 : \lambda = 4.5$ against the alternative hypothesis $H_1 : \lambda \neq 4.5$, based on a single observation of G. The critical region for the test is $G \leqslant 1$ and $G \geqslant 9$.

(i) Find the significance level of the test. [5]

(ii) Given that $\lambda = 5.5$, calculate the probability that the test results in a Type II error. [3]

4 The random variable Y has the distribution $N(\mu, \sigma^2)$. The results of 40 independent observations of Y are summarised by

$$\Sigma y = 3296.0, \quad \Sigma y^2 = 286\,800.40.$$

(i) Calculate unbiased estimates of μ and σ^2. [4]

(ii) Use your answers to part **(i)** to estimate the probability that a single random observation of Y will be less than 60.0. [2]

(iii) Explain whether it is necessary to know that Y is normally distributed in answering part **(i)** of this question. [1]

5 Over a long period the number of visitors per week to a stately home was known to have the distribution $N(500, 100^2)$. After higher car parking charges were introduced, a sample of four randomly chosen weeks gave a mean number of visitors per week of 435. You should assume that the number of visitors per week is still normally distributed with variance 100^2.

(i) Test, at the 10% significance level, whether there is evidence that the mean number of visitors per week has fallen. [7]

(ii) Explain why it is necessary to assume that the distribution of the number of visitors per week (after the introduction of higher charges) is normal in order to carry out the test. [2]

6 The number of house sales per week handled by an estate agent is modelled by the distribution Po(3).

(i) Find the probability that, in one randomly chosen week, the number of sales handled is

(a) greater than 4, [2]

(b) exactly 4. [2]

(ii) Use a suitable approximation to the Poisson distribution to find the probability that, in a year consisting of 50 working weeks, the estate agent handles more than 165 house sales. [5]

(iii) One of the conditions needed for the use of a Poisson model to be valid is that house sales are independent of one another.

(a) Explain, in non-technical language, what you understand by this condition. [1]

(b) State another condition that is needed. [1]

7 A continuous random variable X_1 has probability density function given by

$$\mathrm{f}(x) = \begin{cases} kx & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Find $\mathrm{E}(X_1)$ and $\mathrm{Var}(X_1)$. [5]

(iv) Sketch the graph of $y = \mathrm{f}(x - 1)$. [2]

(v) The continuous random variable X_2 has probability density function $\mathrm{f}(x - 1)$ for all x. Write down the values of $\mathrm{E}(X_2)$ and $\mathrm{Var}(X_2)$. [2]

8 Consultations are taking place as to whether a site currently in use as a car park should be developed as a shopping mall. An agency acting on behalf of a firm of developers claims that at least 65% of the local population are in favour of the development. In a survey of a random sample of 12 members of the local population, 6 are in favour of the development.

(i) Carry out a test, at the 10% significance level, to determine whether the result of the survey is consistent with the claim of the agency. [7]

(ii) A local residents' group claims that no more than 35% of the local population are in favour of the development. Without further calculations, state with a reason what can be said about the claim of the local residents' group. [2]

(iii) A test is carried out, at the 15% significance level, of the agency's claim. The test is based on a random sample of size $2n$, and exactly n of the sample are in favour of the development. Find the smallest possible value of n for which the outcome of the test is to reject the agency's claim. [4]