OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4727**

Further Pure Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The line l_1 passes through the points (0, 0, 10) and (7, 0, 0) and the line l_2 passes through the points (4, 6, 0) and (3, 3, 1). Find the shortest distance between l_1 and l_2. **[7]**

2 A multiplicative group with identity e contains distinct elements a and r, with the properties $r^6 = e$ and $ar = r^5a$.

(i) Prove that $rar = a$. **[2]**

(ii) Prove, by induction or otherwise, that $r^n ar^n = a$ for all positive integers n. **[4]**

3 In this question, w denotes the complex number $\cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$.

(i) Express w^2, w^3 and w^* in polar form, with arguments in the interval $0 \leqslant \theta < 2\pi$. **[4]**

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1 + w, \quad 1 + w + w^2, \quad 1 + w + w^2 + w^3, \quad 1 + w + w^2 + w^3 + w^4$$

are denoted by A, B, C, D, E respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) **[4]**

(iii) Write down a polynomial equation of degree 5 which is satisfied by w. **[1]**

4 **(i)** Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x\cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. (You may quote an appropriate result given in the List of Formulae (MF1).) **[6]**

(ii) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. **[2]**

5 Convergent infinite series C and S are defined by

$$C = 1 + \tfrac{1}{2}\cos\theta + \tfrac{1}{4}\cos 2\theta + \tfrac{1}{8}\cos 3\theta + \ldots ,$$
$$S = \tfrac{1}{2}\sin\theta + \tfrac{1}{4}\sin 2\theta + \tfrac{1}{8}\sin 3\theta + \ldots .$$

(i) Show that $C + \mathrm{i}S = \dfrac{2}{2 - \mathrm{e}^{\mathrm{i}\theta}}$. **[4]**

(ii) Hence show that $C = \dfrac{4 - 2\cos\theta}{5 - 4\cos\theta}$, and find a similar expression for S. **[4]**

6 **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36.$$ **[7]**

(ii) Show that, when x is large and positive, the solution approximates to a linear function, and state its equation. **[2]**

7 A line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. A plane Π passes through the points (1, 3, 5) and (5, 2, 5), and is parallel to l.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. **[4]**

(ii) Find the distance between l and Π. **[4]**

(iii) Find an equation of the line which is the reflection of l in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

8 A set of matrices M is defined by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix},$$

where ω and ω^2 are the complex cube roots of 1. It is given that M is a group under matrix multiplication.

(i) Write down the elements of a subgroup of order 2. **[1]**

(ii) Explain why there is no element X of the group, other than A, which satisfies the equation $X^5 = A$. **[2]**

(iii) By finding BE and EB, verify the closure property for the pair of elements B and E. **[4]**

(iv) Find the inverses of B and E. **[3]**

(v) Determine whether the group M is isomorphic to the group N which is defined as the set of numbers {1, 2, 4, 8, 7, 5} under multiplication modulo 9. Justify your answer clearly. **[3]**