

**ADVANCED GCE**  
**MATHEMATICS**  
Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Monday 24 May 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The line  $l_1$  passes through the points  $(0, 0, 10)$  and  $(7, 0, 0)$  and the line  $l_2$  passes through the points  $(4, 6, 0)$  and  $(3, 3, 1)$ . Find the shortest distance between  $l_1$  and  $l_2$ . [7]

2 A multiplicative group with identity  $e$  contains distinct elements  $a$  and  $r$ , with the properties  $r^6 = e$  and  $ar = r^5a$ .

(i) Prove that  $rar = a$ . [2]

(ii) Prove, by induction or otherwise, that  $r^n ar^n = a$  for all positive integers  $n$ . [4]

3 In this question,  $w$  denotes the complex number  $\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi$ .

(i) Express  $w^2$ ,  $w^3$  and  $w^*$  in polar form, with arguments in the interval  $0 \leq \theta < 2\pi$ . [4]

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1 + w, \quad 1 + w + w^2, \quad 1 + w + w^2 + w^3, \quad 1 + w + w^2 + w^3 + w^4$$

are denoted by  $A, B, C, D, E$  respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) [4]

(iii) Write down a polynomial equation of degree 5 which is satisfied by  $w$ . [1]

4 (i) Use the substitution  $y = xz$  to find the general solution of the differential equation

$$x \frac{dy}{dx} - y = x \cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. (You may quote an appropriate result given in the List of Formulae (MF1).) [6]

(ii) Find the solution of the differential equation for which  $y = \pi$  when  $x = 4$ . [2]

5 Convergent infinite series  $C$  and  $S$  are defined by

$$C = 1 + \frac{1}{2} \cos \theta + \frac{1}{4} \cos 2\theta + \frac{1}{8} \cos 3\theta + \dots,$$

$$S = \frac{1}{2} \sin \theta + \frac{1}{4} \sin 2\theta + \frac{1}{8} \sin 3\theta + \dots$$

(i) Show that  $C + iS = \frac{2}{2 - e^{i\theta}}$ . [4]

(ii) Hence show that  $C = \frac{4 - 2 \cos \theta}{5 - 4 \cos \theta}$ , and find a similar expression for  $S$ . [4]

- 6 (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36. \quad [7]$$

- (ii) Show that, when  $x$  is large and positive, the solution approximates to a linear function, and state its equation. [2]

- 7 A line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ . A plane  $\Pi$  passes through the points  $(1, 3, 5)$  and  $(5, 2, 5)$ , and is parallel to  $l$ .

- (i) Find an equation of  $\Pi$ , giving your answer in the form  $\mathbf{r} \cdot \mathbf{n} = p$ . [4]

- (ii) Find the distance between  $l$  and  $\Pi$ . [4]

- (iii) Find an equation of the line which is the reflection of  $l$  in  $\Pi$ , giving your answer in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ . [4]

- 8 A set of matrices  $M$  is defined by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix},$$

where  $\omega$  and  $\omega^2$  are the complex cube roots of 1. It is given that  $M$  is a group under matrix multiplication.

- (i) Write down the elements of a subgroup of order 2. [1]

- (ii) Explain why there is no element  $X$  of the group, other than  $A$ , which satisfies the equation  $X^5 = A$ . [2]

- (iii) By finding  $BE$  and  $EB$ , verify the closure property for the pair of elements  $B$  and  $E$ . [4]

- (iv) Find the inverses of  $B$  and  $E$ . [3]

- (v) Determine whether the group  $M$  is isomorphic to the group  $N$  which is defined as the set of numbers  $\{1, 2, 4, 8, 7, 5\}$  under multiplication modulo 9. Justify your answer clearly. [3]

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