

**ADVANCED GCE
MATHEMATICS**

Further Pure Mathematics 3

FRIDAY 6 JUNE 2008

4727/01

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

- 1 (a) A cyclic multiplicative group G has order 12. The identity element of G is e and another element is r , with order 12.
- (i) Write down, in terms of e and r , the elements of the subgroup of G which is of order 4. [2]
- (ii) Explain briefly why there is no proper subgroup of G in which two of the elements are e and r . [1]
- (b) A group H has order mnp , where m , n and p are prime. State the possible orders of proper subgroups of H . [2]

- 2 Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$. [7]

- 3 (i) Use the substitution $z = x + y$ to show that the differential equation

$$\frac{dy}{dx} = \frac{x + y + 3}{x + y - 1} \quad (\text{A})$$

may be written in the form $\frac{dz}{dx} = \frac{2(z + 1)}{z - 1}$. [3]

- (ii) Hence find the general solution of the differential equation (A). [4]

- 4 (i) By expressing $\cos \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\cos^5 \theta \equiv \frac{1}{16}(\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta). \quad [5]$$

- (ii) Hence solve the equation $\cos 5\theta + 5 \cos 3\theta + 9 \cos \theta = 0$ for $0 \leq \theta \leq \pi$. [4]

- 5 Two lines have equations

$$\frac{x - k}{2} = \frac{y + 1}{-5} = \frac{z - 1}{-3} \quad \text{and} \quad \frac{x - k}{1} = \frac{y + 4}{-4} = \frac{z}{-2},$$

where k is a constant.

- (i) Show that, for all values of k , the lines intersect, and find their point of intersection in terms of k . [6]

- (ii) For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$. [4]

- 6 The operation \circ on real numbers is defined by $a \circ b = a|b|$.

- (i) Show that \circ is not commutative. [2]

- (ii) Prove that \circ is associative. [4]

- (iii) Determine whether the set of real numbers, under the operation \circ , forms a group. [4]

7 The roots of the equation $z^3 - 1 = 0$ are denoted by 1, ω and ω^2 .

(i) Sketch an Argand diagram to show these roots. [1]

(ii) Show that $1 + \omega + \omega^2 = 0$. [2]

(iii) Hence evaluate

(a) $(2 + \omega)(2 + \omega^2)$, [2]

(b) $\frac{1}{2 + \omega} + \frac{1}{2 + \omega^2}$. [2]

(iv) Hence find a cubic equation, with integer coefficients, which has roots 2, $\frac{1}{2 + \omega}$ and $\frac{1}{2 + \omega^2}$. [4]

8 (i) Find the complementary function of the differential equation

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x. \quad [2]$$

(ii) It is given that $y = p(\ln \sin x) \sin x + qx \cos x$, where p and q are constants, is a particular integral of this differential equation.

(a) Show that $p - 2(p + q) \sin^2 x \equiv 1$. [6]

(b) Deduce the values of p and q . [2]

(iii) Write down the general solution of the differential equation. State the set of values of x , in the interval $0 \leq x \leq 2\pi$, for which the solution is valid, justifying your answer. [3]

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