OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4754(A)/01**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper A

THURSDAY 14 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Express $\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $\sin\theta - 3\cos\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [7]

2 Write down normal vectors to the planes $2x + 3y + 4z = 10$ and $x - 2y + z = 5$.

Hence show that these planes are perpendicular to each other. [4]

3 Fig. 3 shows the curve $y = \ln x$ and part of the line $y = 2$.

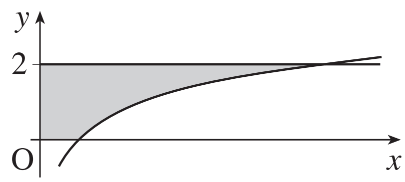

Fig. 3

The shaded region is rotated through 360° about the y-axis.

(i) Show that the volume of the solid of revolution formed is given by $\int_0^2 \pi e^{2y}\,dy$. [3]

(ii) Evaluate this, leaving your answer in an exact form. [3]

4 A curve is defined by parametric equations

$$x = \frac{1}{t} - 1, \quad y = \frac{2+t}{1+t}.$$

Show that the cartesian equation of the curve is $y = \dfrac{3+2x}{2+x}$. [4]

5 Verify that the point $(-1, 6, 5)$ lies on both the lines

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$

Find the acute angle between the lines. [7]

6 Two students are trying to evaluate the integral $\int_1^2 \sqrt{1+e^{-x}}\,dx$.

Sarah uses the trapezium rule with 2 strips, and starts by constructing the following table.

x	1	1.5	2
$\sqrt{1+e^{-x}}$	1.1696	1.1060	1.0655

(i) Complete the calculation, giving your answer to 3 significant figures. [2]

Anish uses a binomial approximation for $\sqrt{1+e^{-x}}$ and then integrates this.

(ii) Show that, provided e^{-x} is suitably small, $\left(1+e^{-x}\right)^{\frac{1}{2}} \approx 1+\frac{1}{2}e^{-x}-\frac{1}{8}e^{-2x}$. [3]

(iii) Use this result to evaluate $\int_1^2 \sqrt{1+e^{-x}}\,dx$ approximately, giving your answer to 3 significant figures. [3]

Section B (36 marks)

7 Data suggest that the number of cases of infection from a particular disease tends to oscillate between two values over a period of approximately 6 months.

(a) Suppose that the number of cases, P thousand, after time t months is modelled by the equation $P = \frac{2}{2 - \sin t}$. Thus, when $t = 0, P = 1$.

(i) By considering the greatest and least values of $\sin t$, write down the greatest and least values of P predicted by this model. [2]

(ii) Verify that P satisfies the differential equation $\frac{dP}{dt} = \frac{1}{2}P^2 \cos t$. [5]

(b) An alternative model is proposed, with differential equation

$$\frac{dP}{dt} = \frac{1}{2}(2P^2 - P)\cos t. \qquad (*)$$

As before, $P = 1$ when $t = 0$.

(i) Express $\frac{1}{P(2P-1)}$ in partial fractions. [4]

(ii) Solve the differential equation (*) to show that

$$\ln\left(\frac{2P-1}{P}\right) = \frac{1}{2}\sin t. \qquad [5]$$

This equation can be rearranged to give $P = \frac{1}{2 - e^{\frac{1}{2}\sin t}}$.

(iii) Find the greatest and least values of P predicted by this model. [4]

8

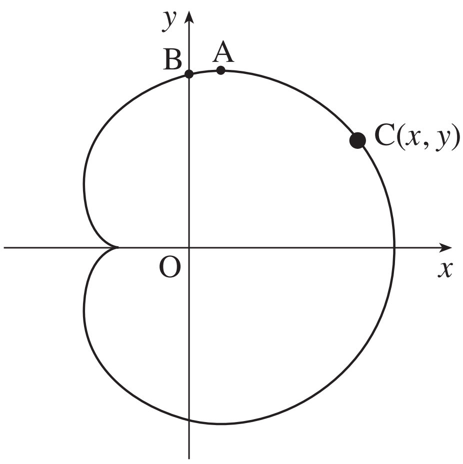

Fig. 8

In a theme park ride, a capsule C moves in a vertical plane (see Fig. 8). With respect to the axes shown, the path of C is modelled by the parametric equations

$$x = 10\cos\theta + 5\cos 2\theta, \quad y = 10\sin\theta + 5\sin 2\theta, \qquad (0 \leqslant \theta < 2\pi),$$

where x and y are in metres.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$.

Verify that $\dfrac{dy}{dx} = 0$ when $\theta = \frac{1}{3}\pi$. Hence find the exact coordinates of the highest point A on the path of C. [6]

(ii) Express $x^2 + y^2$ in terms of θ. Hence show that

$$x^2 + y^2 = 125 + 100\cos\theta. \qquad [4]$$

(iii) Using this result, or otherwise, find the greatest and least distances of C from O. [2]

You are given that, at the point B on the path vertically above O,

$$2\cos^2\theta + 2\cos\theta - 1 = 0.$$

(iv) Using this result, and the result in part (**ii**), find the distance OB. Give your answer to 3 significant figures. [4]

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