

**ADVANCED GCE
MATHEMATICS**

Core Mathematics 3

MONDAY 2 JUNE 2008

4723/01

Morning
Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

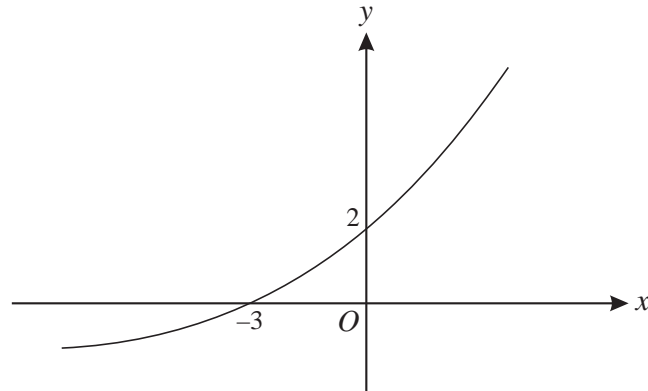
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

- 1 Find the exact solutions of the equation $|4x - 5| = |3x - 5|$. [4]

2



The diagram shows the graph of $y = f(x)$. It is given that $f(-3) = 0$ and $f(0) = 2$. Sketch, on separate diagrams, the following graphs, indicating in each case the coordinates of the points where the graph crosses the axes:

(i) $y = f^{-1}(x)$, [2]

(ii) $y = -2f(x)$. [3]

- 3 Find, in the form $y = mx + c$, the equation of the tangent to the curve

$$y = x^2 \ln x$$

at the point with x -coordinate e . [6]

- 4 The gradient of the curve $y = (2x^2 + 9)^{\frac{5}{2}}$ at the point P is 100.

(i) Show that the x -coordinate of P satisfies the equation $x = 10(2x^2 + 9)^{-\frac{3}{2}}$. [3]

(ii) Show by calculation that the x -coordinate of P lies between 0.3 and 0.4. [3]

(iii) Use an iterative formula, based on the equation in part (i), to find the x -coordinate of P correct to 4 decimal places. You should show the result of each iteration. [3]

- 5 (a) Express $\tan 2\alpha$ in terms of $\tan \alpha$ and hence solve, for $0^\circ < \alpha < 180^\circ$, the equation

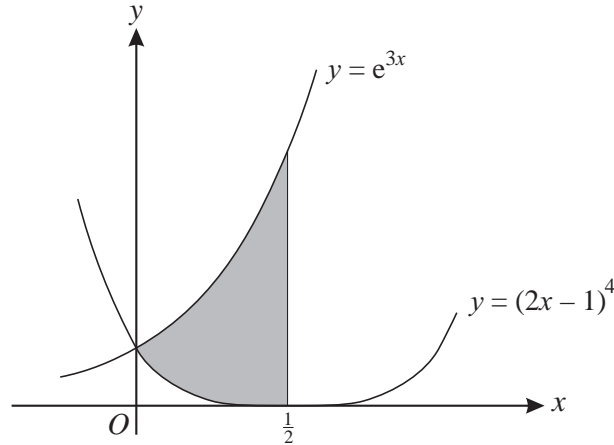
$$\tan 2\alpha \tan \alpha = 8. \quad [6]$$

(b) Given that β is the acute angle such that $\sin \beta = \frac{6}{7}$, find the exact value of

(i) $\operatorname{cosec} \beta$, [1]

(ii) $\cot^2 \beta$. [2]

6



The diagram shows the curves $y = e^{3x}$ and $y = (2x - 1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x -axis. Find the exact volume of the solid produced. [9]

- 7 It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants now is 42. The number of plants is treated as a continuous variable and is denoted by N . The number of years from now is denoted by t .

(i) Two equivalent expressions giving N in terms of t are

$$N = A \times 2^{kt} \quad \text{and} \quad N = Ae^{mt}.$$

Determine the value of each of the constants A , k and m . [4]

(ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures. [2]

(iii) Find the rate at which the number of plants will be increasing at a time 35 years from now. [3]

- 8 The expression $T(\theta)$ is defined for θ in degrees by

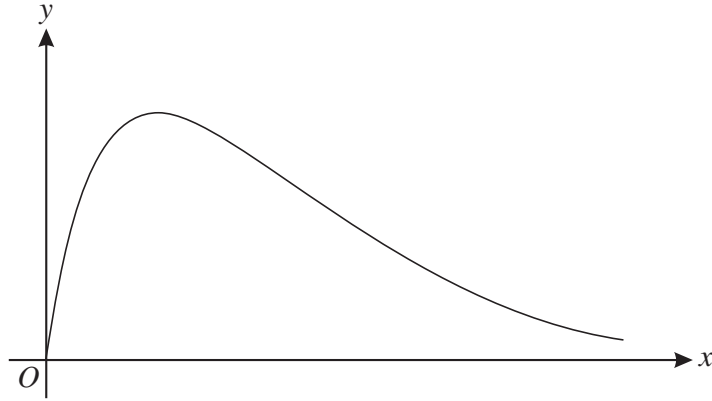
$$T(\theta) = 3 \cos(\theta - 60^\circ) + 2 \cos(\theta + 60^\circ).$$

(i) Express $T(\theta)$ in the form $A \sin \theta + B \cos \theta$, giving the exact values of the constants A and B . [3]

(ii) Hence express $T(\theta)$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the smallest positive value of θ such that $T(\theta) + 1 = 0$. [4]

[Question 9 is printed overleaf.]



The function f is defined for the domain $x \geq 0$ by

$$f(x) = \frac{15x}{x^2 + 5}.$$

The diagram shows the curve with equation $y = f(x)$.

(i) Find the range of f . [6]

(ii) The function g is defined for the domain $x \geq k$ by

$$g(x) = \frac{15x}{x^2 + 5}.$$

Given that g is a one-one function, state the least possible value of k . [1]

(iii) Show that there is no point on the curve $y = g(x)$ at which the gradient is -1 . [4]