OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

# MATHEMATICS

**4726**

Further Pure Mathematics 2

Candidates answer on the answer booklet.

**OCR supplied materials:**
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Monday 20 June 2011**
**Morning**

**Duration:** 1 hour 30 minutes

* 4 7 2 6 *

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**1** Express $\dfrac{2x+3}{(x+3)(x^2+9)}$ in partial fractions. **[5]**

**2** A curve has equation $y = \dfrac{x^2-6x-5}{x-2}$.

**(i)** Find the equations of the asymptotes. **[3]**

**(ii)** Show that $y$ can take all real values. **[4]**

**3** It is given that $\mathrm{F}(x) = 2 + \ln x$. The iteration $x_{n+1} = \mathrm{F}(x_n)$ is to be used to find a root, $\alpha$, of the equation $x = 2 + \ln x$.

**(i)** Taking $x_1 = 3.1$, find $x_2$ and $x_3$, giving your answers correct to 5 decimal places. **[2]**

**(ii)** The error $e_n$ is defined by $e_n = \alpha - x_n$. Given that $\alpha = 3.146\,19$, correct to 5 decimal places, use the values of $e_2$ and $e_3$ to make an estimate of $\mathrm{F}'(\alpha)$ correct to 3 decimal places. State the true value of $\mathrm{F}'(\alpha)$ correct to 4 decimal places. **[3]**

**(iii)** Illustrate the iteration by drawing a sketch of $y = x$ and $y = \mathrm{F}(x)$, showing how the values of $x_n$ approach $\alpha$. State whether the convergence is of the 'staircase' or 'cobweb' type. **[3]**

**4** A curve $C$ has the cartesian equation $x^3 + y^3 = axy$, where $x \geqslant 0$, $y \geqslant 0$ and $a > 0$.

**(i)** Express the polar equation of $C$ in the form $r = \mathrm{f}(\theta)$ and state the limits between which $\theta$ lies. **[3]**

The line $\theta = \alpha$ is a line of symmetry of $C$.

**(ii)** Find and simplify an expression for $\mathrm{f}\left(\frac{1}{2}\pi - \theta\right)$ and hence explain why $\alpha = \frac{1}{4}\pi$. **[3]**

**(iii)** Find the value of $r$ when $\theta = \frac{1}{4}\pi$. **[1]**

**(iv)** Sketch the curve $C$. **[2]**

**5** **(i)** Prove that, if $y = \sin^{-1} x$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{1-x^2}}$. **[3]**

**(ii)** Find the Maclaurin series for $\sin^{-1} x$, up to and including the term in $x^3$. **[5]**

**(iii)** Use the result of part **(ii)** and the Maclaurin series for $\ln(1+x)$ to find the Maclaurin series for $(\sin^{-1} x)\ln(1+x)$, up to and including the term in $x^4$. **[4]**

**6** It is given that $I_n = \displaystyle\int_0^1 x^n (1-x)^{\frac{3}{2}}\,\mathrm{d}x$, for $n \geqslant 0$.

**(i)** Show that $I_n = \dfrac{2n}{2n+5} I_{n-1}$, for $n \geqslant 1$. **[6]**

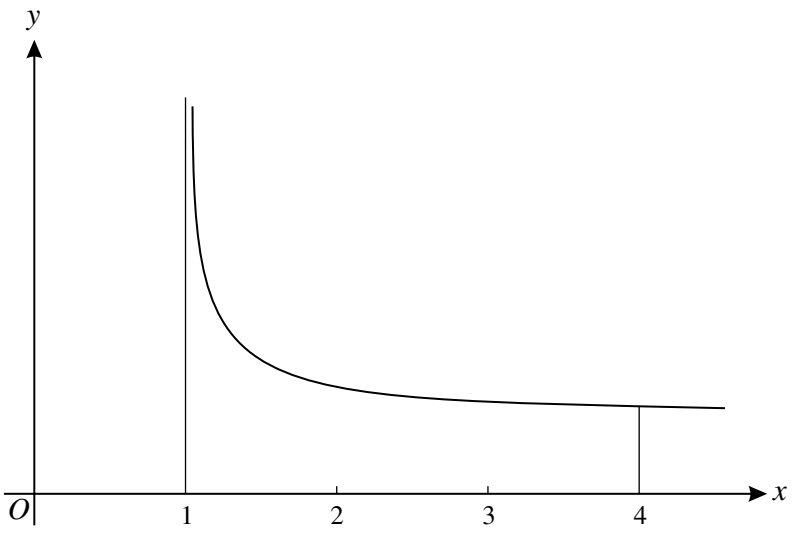
**(ii)** Hence find the exact value of $I_3$. **[4]**

**7** **(i)** Sketch the graph of $y = \tanh x$ and state the value of the gradient when $x = 0$. On the same axes, sketch the graph of $y = \tanh^{-1} x$. Label each curve and give the equations of the asymptotes. **[4]**

**(ii)** Find $\int_0^k \tanh x \, \mathrm{d}x$, where $k > 0$. **[2]**

**(iii)** Deduce, or show otherwise, that $\int_0^{\tanh k} \tanh^{-1} x \, \mathrm{d}x = k \tanh k - \ln(\cosh k)$. **[4]**

**8** **(i)** Use the substitution $x = \cosh^2 u$ to find $\int \sqrt{\frac{x}{x-1}} \, \mathrm{d}x$, giving your answer in the form $\mathrm{f}(x) + \ln\big(\mathrm{g}(x)\big)$. **[7]**




**(ii)** Hence calculate the exact area of the region between the curve $y = \sqrt{\frac{x}{x-1}}$, the $x$-axis and the lines $x = 1$ and $x = 4$ (see diagram). **[1]**

**(iii)** What can you say about the volume of the solid of revolution obtained when the region defined in part **(ii)** is rotated completely about the $x$-axis? Justify your answer. **[3]**