

**ADVANCED GCE**  
**MATHEMATICS**  
Probability & Statistics 4

**4735**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Thursday 24 June 2010**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 For the variables  $A$  and  $B$ , it is given that  $\text{Var}(A) = 9$ ,  $\text{Var}(B) = 6$  and  $\text{Var}(2A - 3B) = 18$ .

(i) Find  $\text{Cov}(A, B)$ . [3]

(ii) State with a reason whether  $A$  and  $B$  are independent. [1]

2 The probability generating function of the discrete random variable  $X$  is  $\frac{e^{4t^2}}{e^4}$ . Find

(i)  $E(X)$ , [3]

(ii)  $P(X = 2)$ . [3]

3  $X_1$  and  $X_2$  are continuous random variables. Random samples of 5 observations of  $X_1$  and 6 observations of  $X_2$  are taken. No two observations are equal. The 11 observations are ranked, lowest first, and the sum of the ranks of the observations of  $X_1$  is denoted by  $R$ .

(i) Assuming that all rankings are equally likely, show that  $P(R \leq 17) = \frac{2}{231}$ . [5]

The marks of 5 randomly chosen students from School  $A$  and 6 randomly chosen students from School  $B$ , who took the same examination, achieving different marks, were ranked. The rankings are shown in the table.

Rank	1	2	3	4	5	6	7	8	9	10	11
School	$A$	$A$	$A$	$B$	$A$	$A$	$B$	$B$	$B$	$B$	$B$

(ii) For a Wilcoxon rank-sum test, obtain the exact smallest significance level for which there is evidence of a difference in performance at the two schools. [2]

4 The moment generating function of a continuous random variable  $Y$ , which has a  $\chi^2$  distribution with  $n$  degrees of freedom, is  $(1 - 2t)^{-\frac{1}{2}n}$ , where  $0 \leq t < \frac{1}{2}$ .

(i) Find  $E(Y)$  and  $\text{Var}(Y)$ . [5]

For the case  $n = 1$ , the sum of 60 independent observations of  $Y$  is denoted by  $S$ .

(ii) Write down the moment generating function of  $S$  and hence identify the distribution of  $S$ . [2]

(iii) Use a normal approximation to estimate  $P(S \geq 70)$ . [3]

5 In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries  $x$ , in thousands of pounds, of 50 randomly chosen employees were obtained.

(i) The values  $|x - 19.5|$  were calculated and ranked. No two values of  $x$  were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of  $(x - 19.5)$  was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. [10]

(ii) If the assumption you stated in part (i) does not hold, what test could have been used? [1]

- 6 Nuts and raisins occur in randomly chosen squares of a particular brand of chocolate. The numbers of nuts and raisins are denoted by  $N$  and  $R$  respectively and the joint probability distribution of  $N$  and  $R$  is given by

$$f(n, r) = \begin{cases} c(n + 2r) & n = 0, 1, 2 \text{ and } r = 0, 1, 2, \\ 0 & \text{otherwise,} \end{cases}$$

where  $c$  is a constant.

- (i) Find the value of  $c$ . [3]
- (ii) Find the probability that there is exactly one nut in a randomly chosen square. [2]
- (iii) Find the probability that the total number of nuts and raisins in a randomly chosen square is more than 2. [2]
- (iv) For squares in which there are 2 raisins, find the mean number of nuts. [4]
- (v) Determine whether  $N$  and  $R$  are independent. [2]
- 7 The continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} \frac{x}{2\theta^2} & 0 \leq x \leq 2\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where  $\theta$  is an unknown positive constant.

- (i) Find  $E(X^n)$ , where  $n \neq -2$ , and hence write down the value of  $E(X)$ . [3]
- (ii) Find
- (a)  $\text{Var}(X)$ ,
- (b)  $\text{Var}(X^2)$ . [4]
- (iii) Find  $E(X_1 + X_2 + X_3)$  and  $E(X_1^2 + X_2^2 + X_3^2)$ , where  $X_1, X_2$  and  $X_3$  are independent observations of  $X$ . Hence construct unbiased estimators,  $T_1$  and  $T_2$ , of  $\theta$  and  $\text{Var}(X)$  respectively, which are based on  $X_1, X_2$  and  $X_3$ . [6]
- (iv) Find  $\text{Var}(T_2)$ . [2]
- 8 For the events  $L$  and  $M$ ,  $P(L | M) = 0.2$ ,  $P(M | L) = 0.4$  and  $P(M) = 0.6$ .
- (i) Find  $P(L)$  and  $P(L' \cup M')$ . [3]
- (ii) Given that, for the event  $N$ ,  $P(N | (L \cap M)) = 0.3$ , find  $P(L' \cup M' \cup N')$ . [3]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity. For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.