RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS (MEI)

4763/01

Mechanics 3

MONDAY 21 MAY 2007

Morning

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

1 **(a)** **(i)** Write down the dimensions of the following quantities.

Velocity
Acceleration
Force
Density (which is mass per unit volume)
Pressure (which is force per unit area) [5]

For a fluid with constant density ρ, the velocity v, pressure P and height h at points on a streamline are related by Bernoulli's equation

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

where g is the acceleration due to gravity.

(ii) Show that the left-hand side of Bernoulli's equation is dimensionally consistent. [4]

(b) In a wave tank, a float is performing simple harmonic motion with period 3.49 s in a vertical line. The height of the float above the bottom of the tank is h m at a time t s. When $t = 0$, the height has its maximum value. The value of h varies between 1.6 and 2.2.

(i) Sketch a graph showing how h varies with t. [2]

(ii) Express h in terms of t. [4]

(iii) Find the magnitude and direction of the acceleration of the float when $h = 1.7$. [3]

2 A fixed hollow sphere with centre O has an inside radius of 2.7 m. A particle P of mass 0.4 kg moves on the smooth inside surface of the sphere.

At first, P is moving in a horizontal circle with constant speed, and OP makes a constant angle of 60° with the vertical (see Fig. 2.1).

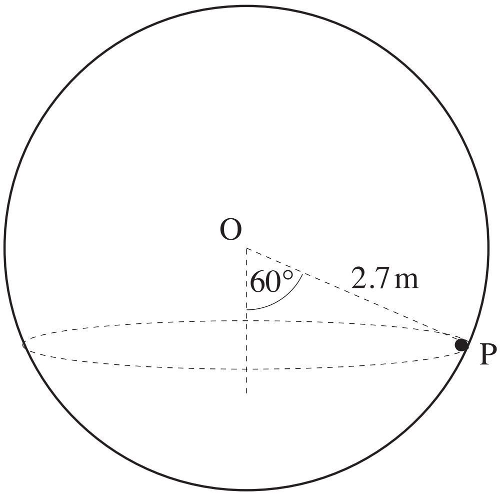

Fig. 2.1

(i) Find the normal reaction acting on P. [2]

(ii) Find the speed of P. [4]

The particle P is now placed at the lowest point of the sphere and is given an initial horizontal speed of 9 m s^{-1}. It then moves in part of a vertical circle. When OP makes an angle θ with the upward vertical and P is still in contact with the sphere, the speed of P is $v \text{ m s}^{-1}$ and the normal reaction acting on P is R N (see Fig. 2.2).

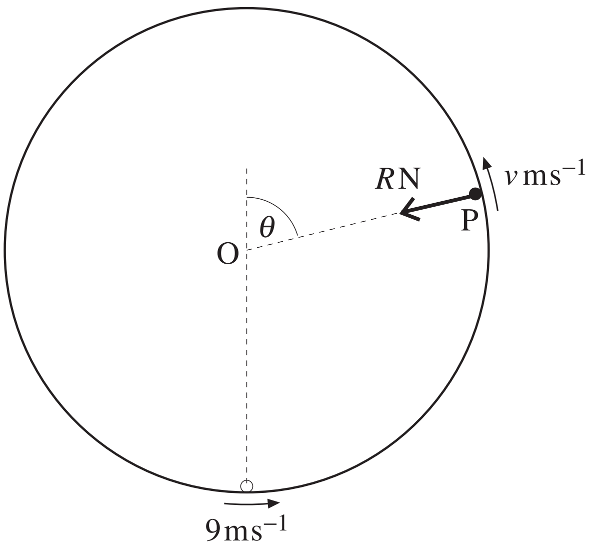

Fig. 2.2

(iii) Find v^2 in terms of θ. [3]

(iv) Show that $R = 4.16 - 11.76\cos\theta$. [5]

(v) Find the speed of P at the instant when it leaves the surface of the sphere. [4]

[Turn over

3 A light elastic string has natural length 1.2 m and stiffness $637\,\mathrm{N\,m^{-1}}$.

(i) The string is stretched to a length of 1.3 m. Find the tension in the string and the elastic energy stored in the string. [3]

One end of this string is attached to a fixed point A. The other end is attached to a heavy ring R which is free to move along a smooth vertical wire. The shortest distance from A to the wire is 1.2 m (see Fig. 3).

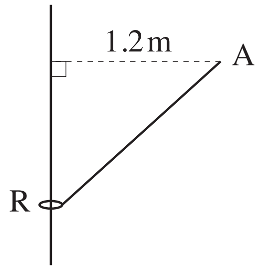

Fig. 3

The ring is in equilibrium when the length of the string AR is 1.3 m.

(ii) Show that the mass of the ring is 2.5 kg. [4]

The ring is given an initial speed $u\,\mathrm{m\,s^{-1}}$ vertically downwards from its equilibrium position. It first comes to rest, instantaneously, in the position where the length of AR is 1.5 m.

(iii) Find u. [7]

(iv) Determine whether the ring will rise above the level of A. [4]

4 **(a)** The region bounded by the curve $y = x^3$ for $0 \leqslant x \leqslant 2$, the x-axis and the line $x = 2$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [8]

(b) The region bounded by the circular arc $y = \sqrt{4 - x^2}$ for $1 \leqslant x \leqslant 2$, the x-axis and the line $x = 1$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution, as shown in Fig. 4.1.

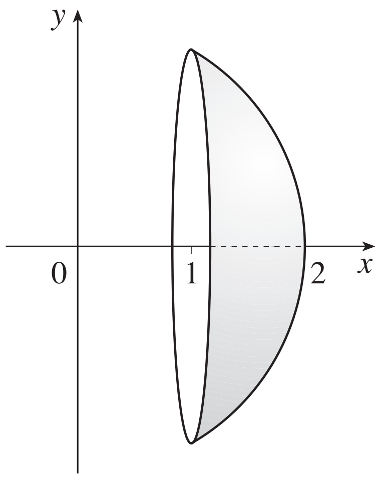

Fig. 4.1

(i) Show that the x-coordinate of the centre of mass of this solid of revolution is 1.35. [6]

This solid is placed on a rough horizontal surface, with its flat face in a vertical plane. It is held in equilibrium by a light horizontal string attached to its highest point and perpendicular to its flat face, as shown in Fig. 4.2.

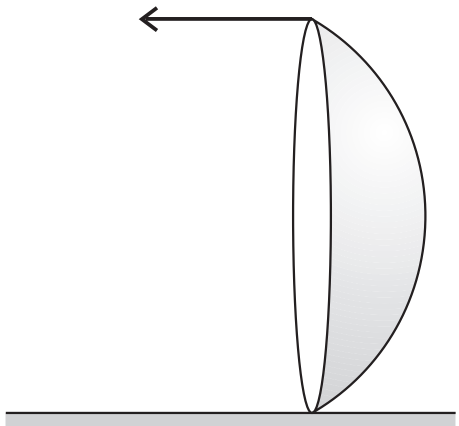

Fig. 4.2

(ii) Find the least possible coefficient of friction between the solid and the horizontal surface. [4]

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