OCR
RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS

4733/01 Probability & Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4733/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A random variable has the distribution $B(n, p)$. It is required to test $H_0: p = \frac{2}{3}$ against $H_1: p < \frac{2}{3}$ at a significance level as close to 1% as possible, using a sample of size $n = 8$, 9 or 10. Use tables to find which value of n gives such a test, stating the critical region for the test and the corresponding significance level. **[4]**

2 A random variable C has the distribution $N(\mu, \sigma^2)$. A random sample of 10 observations of C is obtained, and the results are summarised as

$$n = 10, \Sigma c = 380, \Sigma c^2 = 14602.$$

(i) Calculate unbiased estimates of μ and σ^2. **[4]**

(ii) Hence calculate an estimate of the probability that $C > 40$. **[2]**

3 A factory produces 9000 music DVDs each day. A random sample of 100 such DVDs is obtained.

(i) Explain how to obtain this sample using random numbers. **[3]**

(ii) Given that 24% of the DVDs produced by the factory are classical, use a suitable approximation to find the probability that, in the sample of 100 DVDs, fewer than 20 are classical. **[5]**

4 A continuous random variable X has probability density function

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants.

(i) State what the letter x represents. **[1]**

(ii) Find k in terms of a. **[2]**

(iii) Find $\text{Var}(X)$ in terms of a. **[6]**

5 In a mine, a deposit of the substance *pitchblende* emits radioactive particles. The number of particles emitted has a Poisson distribution with mean 70 particles per second. The warning level is reached if the total number of particles emitted in one minute is more than 4350.

(i) A one-minute period is chosen at random. Use a suitable approximation to show that the probability that the warning level is reached during this period is 0.01, correct to 2 decimal places. You should calculate the answer correct to 4 decimal places. **[5]**

(ii) Use a suitable approximation to find the probability that in 30 one-minute periods the warning level is reached on at least 4 occasions. (You should use the given rounded value of 0.01 from part **(i)** in your calculation.) **[3]**

6 Gordon is a cricketer. Over a long period he knows that his population mean score, in number of runs per innings, is 28, and the population standard deviation is 12. In a new season he adopts a different batting style and he finds that in 30 innings using this style his mean score is 28.98.

(i) Stating a necessary assumption, test at the 5% significance level whether his population mean score has increased. **[8]**

(ii) Explain whether it was necessary to use the Central Limit Theorem in part **(i)**. **[2]**

7 The continuous random variable X has the distribution $\mathrm{N}(\mu, \sigma^2)$. The mean of a random sample of n observations of X is denoted by $\overline{X}$. It is given that $\mathrm{P}(\overline{X} < 35.0) = 0.9772$ and $\mathrm{P}(\overline{X} < 20.0) = 0.1587$.

(i) Obtain a formula for σ in terms of n. **[5]**

Two students are discussing this question. Aidan says "If you were told another probability, for instance $\mathrm{P}(\overline{X} > 32) = 0.1$, you could work out the value of σ." Binya says, "No, the value of $\mathrm{P}(\overline{X} > 32)$ is fixed by the information you know already."

(ii) State which of Aidan and Binya is right. If you think that Aidan is right, calculate the value of σ given that $\mathrm{P}(\overline{X} > 32) = 0.1$. If you think that Binya is right, calculate the value of $\mathrm{P}(\overline{X} > 32)$. **[4]**

8 In a large city the number of traffic lights that fail in one day of 24 hours is denoted by Y. It may be assumed that failures occur randomly.

(i) Explain what the statement "failures occur randomly" means. **[1]**

(ii) State, in context, two different conditions that must be satisfied if Y is to be modelled by a Poisson distribution, and for each condition explain whether you think it is likely to be met in this context. **[4]**

(iii) For this part you may assume that Y is well modelled by the distribution $\mathrm{Po}(\lambda)$. It is given that $\mathrm{P}(Y = 7) = \mathrm{P}(Y = 8)$. Use an algebraic method to calculate the value of λ and hence calculate the corresponding value of $\mathrm{P}(Y = 7)$. **[5]**

9 The random variable A has the distribution $\mathrm{B}(30, p)$. A test is carried out of the hypotheses H_0: $p = 0.6$ against H_1: $p < 0.6$. The critical region is $A \leqslant 13$.

(i) State the probability that H_0 is rejected when $p = 0.6$. **[1]**

(ii) Find the probability that a Type II error occurs when $p = 0.5$. **[2]**

(iii) It is known that on average $p = 0.5$ on one day in five, and on other days the value of p is 0.6. On each day two tests are carried out. If the result of the first test is that H_0 is rejected, the value of p is adjusted if necessary, to ensure that $p = 0.6$ for the rest of the day. Otherwise the value of p remains the same as for the first test. Calculate the probability that the result of the second test is to reject H_0. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.