

**Monday 14 January 2013 – Morning**

**A2 GCE MATHEMATICS**

**4726/01 Further Pure Mathematics 2**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4726/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Express  $\frac{5x}{(x-1)(x^2+4)}$  in partial fractions. [5]
- 2 The equation of a curve is  $y = \frac{x^2-3}{x-1}$ .
- (i) Find the equations of the asymptotes of the curve. [3]
- (ii) Write down the coordinates of the points where the curve cuts the axes. [1]
- (iii) Show that the curve has no stationary points. [3]
- (iv) Sketch the curve and the asymptotes. [3]
- 3 By first expressing  $\cosh x$  and  $\sinh x$  in terms of exponentials, solve the equation
- $$3 \cosh x - 4 \sinh x = 7,$$
- giving your answer in an exact logarithmic form. [6]
- 4 You are given that  $I_n = \int_0^1 x^n e^{2x} dx$  for  $n \geq 0$ .
- (i) Show that  $I_n = \frac{1}{2}e^2 - \frac{1}{2}nI_{n-1}$  for  $n \geq 1$ . [4]
- (ii) Find  $I_3$  in terms of e. [4]
- 5 You are given that  $f(x) = e^{-x} \sin x$ .
- (i) Find  $f(0)$  and  $f'(0)$ . [3]
- (ii) Show that  $f''(x) = -2f'(x) - 2f(x)$  and hence, or otherwise, find  $f''(0)$ . [4]
- (iii) Find a similar expression for  $f'''(x)$  and hence, or otherwise, find  $f'''(0)$ . [2]
- (iv) Find the Maclaurin series for  $f(x)$  up to and including the term in  $x^3$ . [2]

6 By first completing the square, find  $\int_0^1 \frac{1}{\sqrt{x^2 + 4x + 8}} dx$ , giving your answer in an exact logarithmic form. [6]

7 A curve has polar equation  $r = 5 \sin 2\theta$  for  $0 \leq \theta \leq \frac{1}{2}\pi$ .

(i) Sketch the curve, indicating the line of symmetry and stating the polar coordinates of the point  $P$  on the curve which is furthest away from the pole. [4]

(ii) Calculate the area enclosed by the curve. [3]

(iii) Find the cartesian equation of the tangent to the curve at  $P$ . [3]

(iv) Show that a cartesian equation of the curve is  $(x^2 + y^2)^3 = (10xy)^2$ . [3]

8 It is required to solve the equation  $\ln(x - 1) - x + 3 = 0$ .

You are given that there are two roots,  $\alpha$  and  $\beta$ , where  $1.1 < \alpha < 1.2$  and  $4.1 < \beta < 4.2$ .

(i) The root  $\beta$  can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with  $x_1 = 4.15$ , find  $\beta$  correct to 3 decimal places. Show all your working. [2]

(b) Explain with the aid of a sketch why this iterative formula will not converge to  $\alpha$  whatever initial value is taken. [3]

(ii) (a) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{3 - 2x_n - (x_n - 1)\ln(x_n - 1)}{2 - x_n}. \quad [5]$$

(b) Use this formula with  $x_1 = 1.2$  to find  $\alpha$  correct to 3 decimal places. [3]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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