

**Wednesday 23 January 2013 – Morning**

**AS GCE MATHEMATICS**

**4725/01 Further Pure Mathematics 1**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

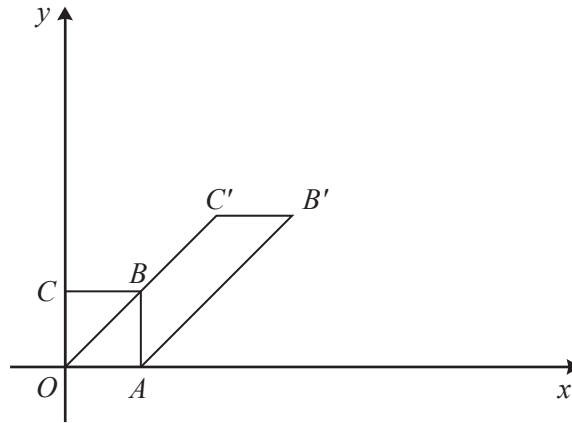
- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$ , where  $a \neq \frac{1}{4}$ , and  $\mathbf{I}$  denotes the  $2 \times 2$  identity matrix. Find
- (i)  $2\mathbf{A} - 3\mathbf{I}$ , [3]
- (ii)  $\mathbf{A}^{-1}$ . [2]
- 2 Find  $\sum_{r=1}^n (r-1)(r+1)$ , giving your answer in a fully factorised form. [6]
- 3 The complex number  $2 - i$  is denoted by  $z$ .
- (i) Find  $|z|$  and  $\arg z$ . [2]
- (ii) Given that  $az + bz^* = 4 - 8i$ , find the values of the real constants  $a$  and  $b$ . [5]
- 4 The quadratic equation  $x^2 + x + k = 0$  has roots  $\alpha$  and  $\beta$ .
- (i) Use the substitution  $x = 2u + 1$  to obtain a quadratic equation in  $u$ . [2]
- (ii) Hence, or otherwise, find the value of  $\left(\frac{\alpha-1}{2}\right)\left(\frac{\beta-1}{2}\right)$  in terms of  $k$ . [2]
- 5 By using the determinant of an appropriate matrix, find the values of  $\lambda$  for which the simultaneous equations
- $$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$
- do not have a unique solution for  $x$ ,  $y$  and  $z$ . [6]

6



The diagram shows the unit square  $OABC$ , and its image  $OAB'C'$  after a transformation. The points have the following coordinates:  $A(1, 0)$ ,  $B(1, 1)$ ,  $C(0, 1)$ ,  $B'(3, 2)$  and  $C'(2, 2)$ .

(i) Write down the matrix,  $\mathbf{X}$ , for this transformation. [2]

(ii) The transformation represented by  $\mathbf{X}$  is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. [6]

(iii) Find the matrix that represents transformation Q followed by transformation P. [2]

7 (i) Sketch on a single Argand diagram the loci given by

(a)  $|z| = 2$ , [2]

(b)  $\arg(z - 3 - i) = \pi$ . [3]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leq 2 \text{ and } 0 \leq \arg(z - 3 - i) \leq \pi. \quad [2]$$

8 (i) Show that  $\frac{1}{r} - \frac{3}{r+1} + \frac{2}{r+2} \equiv \frac{2-r}{r(r+1)(r+2)}$ . [2]

(ii) Hence show that  $\sum_{r=1}^n \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$ . [5]

(iii) Find the value of  $\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$ . [2]

- 9 (i) Show that  $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$ . [3]
- (ii) It is given that  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the cubic equation  $x^3 + px^2 - 4x + 3 = 0$ , where  $p$  is a constant. Find the value of  $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$  in terms of  $p$ . [5]
- 10 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 2$  and  $u_{n+1} = \frac{u_n}{1 + u_n}$  for  $n \geq 1$ .
- (i) Find  $u_2$  and  $u_3$ , and show that  $u_4 = \frac{2}{7}$ . [3]
- (ii) Hence suggest an expression for  $u_n$ . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.