

**Friday 18 January 2013 – Afternoon**

**A2 GCE MATHEMATICS**

**4724/01** Core Mathematics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4724/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Find  $\int x \cos 3x \, dx$ . [4]
- 2 Find the first three terms in the expansion of  $(9 - 16x)^{\frac{3}{2}}$  in ascending powers of  $x$ , and state the set of values for which this expansion is valid. [5]
- 3 The equation of a curve is  $xy^2 = x^2 + 1$ . Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ , and hence find the coordinates of the stationary points on the curve. [7]
- 4 The equations of two lines are
- $$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$
- (i) Show that these lines meet, and find the coordinates of the point of intersection. [5]
- (ii) Find the acute angle between these lines. [3]
- 5 The parametric equations of a curve are
- $$x = 2 + 3 \sin \theta \quad \text{and} \quad y = 1 - 2 \cos \theta \quad \text{for} \quad 0 \leq \theta \leq \frac{1}{2}\pi.$$
- (i) Find the coordinates of the point on the curve where the gradient is  $\frac{1}{2}$ . [5]
- (ii) Find the cartesian equation of the curve. [2]
- 6 Use the substitution  $u = 2x + 1$  to evaluate  $\int_0^{\frac{1}{2}} \frac{4x - 1}{(2x + 1)^5} \, dx$ . [7]
- 7 (i) Given that  $y = \ln(1 + \sin x) - \ln(\cos x)$ , show that  $\frac{dy}{dx} = \frac{1}{\cos x}$ . [4]
- (ii) Using this result, evaluate  $\int_0^{\frac{1}{5}\pi} \sec x \, dx$ , giving your answer as a single logarithm. [3]
- 8 The points  $A(3, 2, 1)$ ,  $B(5, 4, -3)$ ,  $C(3, 17, -4)$  and  $D(1, 6, 3)$  form a quadrilateral  $ABCD$ .
- (i) Show that  $AB = AD$ . [2]
- (ii) Find a vector equation of the line through  $A$  and the mid-point of  $BD$ . [3]
- (iii) Show that  $C$  lies on the line found in part (ii). [1]
- (iv) What type of quadrilateral is  $ABCD$ ? [1]

- 9 The temperature of a freezer is  $-20^{\circ}\text{C}$ . A container of a liquid is placed in the freezer. The rate at which the temperature,  $\theta^{\circ}\text{C}$ , of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time  $t$  is in minutes and  $k$  is a positive constant.

- (i) Express  $\theta$  in terms of  $t$ ,  $k$  and an arbitrary constant. [3]

Initially the temperature of the liquid in the container is  $40^{\circ}\text{C}$  and, at this instant, the liquid is cooling at a rate of  $3^{\circ}\text{C}$  per minute. The liquid freezes at  $0^{\circ}\text{C}$ .

- (ii) Find the value of  $k$  and find also the time it takes (to the nearest minute) for the liquid to freeze. [5]

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is  $90^{\circ}\text{C}$ . After 19 minutes its temperature is  $0^{\circ}\text{C}$ .

- (iii) Without any further calculation, explain what you can deduce about the value of  $k$  in this case. [1]

- 10 (i) Use algebraic division to express  $\frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$  in the form  $Ax + B + \frac{Cx + D}{x^2 - x - 6}$ , where  $A$ ,  $B$ ,  $C$  and  $D$  are constants. [4]

- (ii) Hence find  $\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6} dx$ , giving your answer in the form  $a + \ln b$ . [7]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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