

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS

4723 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4723
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Solve the inequality $|2x - 5| > |x + 1|$. [5]
- 2 It is given that $p = e^{280}$ and $q = e^{300}$.
- (i) Use logarithm properties to show that $\ln\left(\frac{ep^2}{q}\right) = 261$. [3]
- (ii) Find the smallest integer n which satisfies the inequality $5^n > pq$. [3]
- 3 It is given that θ is the acute angle such that $\sec\theta \sin\theta = 36 \cot\theta$.
- (i) Show that $\tan\theta = 6$. [3]
- (ii) Hence, using an appropriate formula in each case, find the exact value of
- (a) $\tan(\theta - 45^\circ)$, [2]
- (b) $\tan 2\theta$. [2]
- 4 (a) Show that $\int_0^4 \frac{18}{\sqrt{6x+1}} dx = 24$. [4]
- (b) Find $\int_0^1 (e^x + 2)^2 dx$, giving your answer in terms of e . [4]
- 5 (i) It is given that k is a positive constant. By sketching the graphs of
- $$y = 14 - x^2 \quad \text{and} \quad y = k \ln x$$
- on a single diagram, show that the equation
- $$14 - x^2 = k \ln x$$
- has exactly one real root. [3]
- (ii) The real root of the equation $14 - x^2 = 3 \ln x$ is denoted by α .
- (a) Find by calculation the pair of consecutive integers between which α lies. [3]
- (b) Use the iterative formula $x_{n+1} = \sqrt{14 - 3 \ln x_n}$, with a suitable starting value, to find α . Show the result of each iteration, and give α correct to 2 decimal places. [4]

- 6 The volume, $V \text{ m}^3$, of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8,$$

where $h \text{ m}$ is the depth of the liquid.

- (i) Find the value of $\frac{dV}{dh}$ when $h = 0.6$, giving your answer correct to 2 decimal places. [4]
- (ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m , the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m . [3]
- 7 The function f is defined for all real values of x by $f(x) = 2x + 5$. The function g is defined for all real values of x and is such that $g^{-1}(x) = \sqrt[3]{x - a}$, where a is a constant. It is given that $fg^{-1}(12) = 9$. Find the value of a and hence solve the equation $gf(x) = 68$. [7]

- 8 (i) Express $3 \sin \theta + 4 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(ii) Hence

- (a) solve the equation $3 \sin \theta + 4 \cos \theta + 1 = 0$, giving all solutions for which $-180^\circ < \theta < 180^\circ$, [4]

- (b) find the values of the positive constants k and c such that

$$-37 \leq k(3 \sin \theta + 4 \cos \theta) + c \leq 43$$

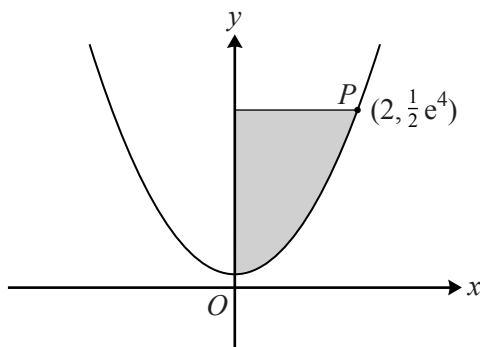
for all values of θ . [4]

- 9 (i) Show that the derivative with respect to y of

$$y \ln(2y) - y$$

is $\ln(2y)$. [3]

(ii)



The diagram shows the curve with equation $y = \frac{1}{2}e^{x^2}$. The point $P(2, \frac{1}{2}e^4)$ lies on the curve. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{1}{2}e^4$. Find the exact volume of the solid produced when the shaded region is rotated completely about the y -axis. [6]

- (iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$ is rotated completely about the y -axis. [2]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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