

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

THURSDAY 21 JUNE 2012: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4753/01

MEI Examination Formulae and Tables (MF2)

Insert for question 4 (inserted)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

SECTION A (36 marks)

- 1 Show that $\int_1^2 \frac{1}{\sqrt{3x-2}} dx = \frac{2}{3}$. [5]
- 2 Solve the inequality $|2x + 1| > 4$. [3]
- 3 Find the gradient at the point $(0, \ln 2)$ on the curve with equation $e^{2y} = 5 - e^{-x}$. [4]
- 4 Fig. 4 below shows the curve $y = f(x)$, where $f(x) = \sqrt{1 - 9x^2}$, $-a \leq x \leq a$.

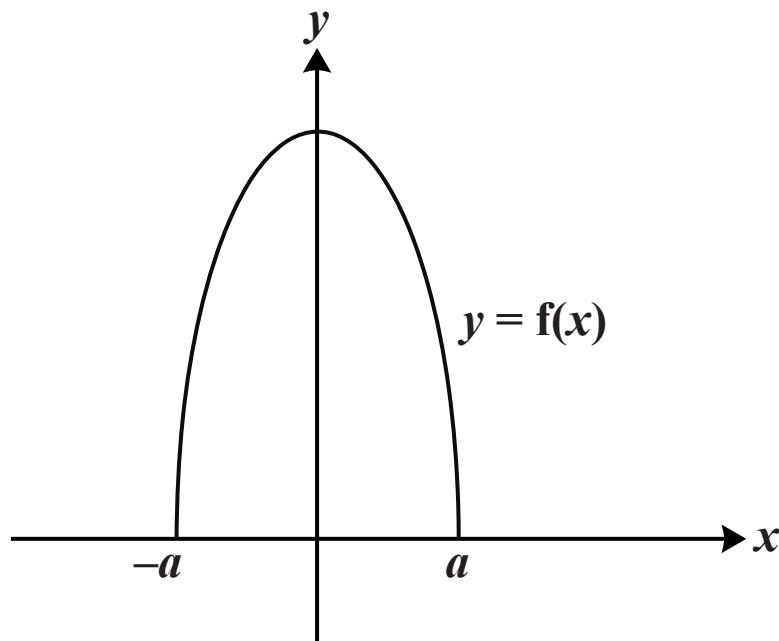


Fig. 4

- (i) Find the value of a . [2]
- (ii) Write down the range of $f(x)$. [1]
- (iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. You may use the insert provided. [3]

5 A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

- (i)** Calculate the population when $t = 0$, and the long-term population, given by this model. [3]
- (ii)** Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k . [3]

- 6 Fig. 6 below shows the curve $y = f(x)$, where $f(x) = 2\arcsin x$, $-1 \leq x \leq 1$.

Fig. 6 also shows the curve $y = g(x)$, where $g(x)$ is the inverse function of $f(x)$.

P is the point on the curve $y = f(x)$ with x -coordinate $\frac{1}{2}$.

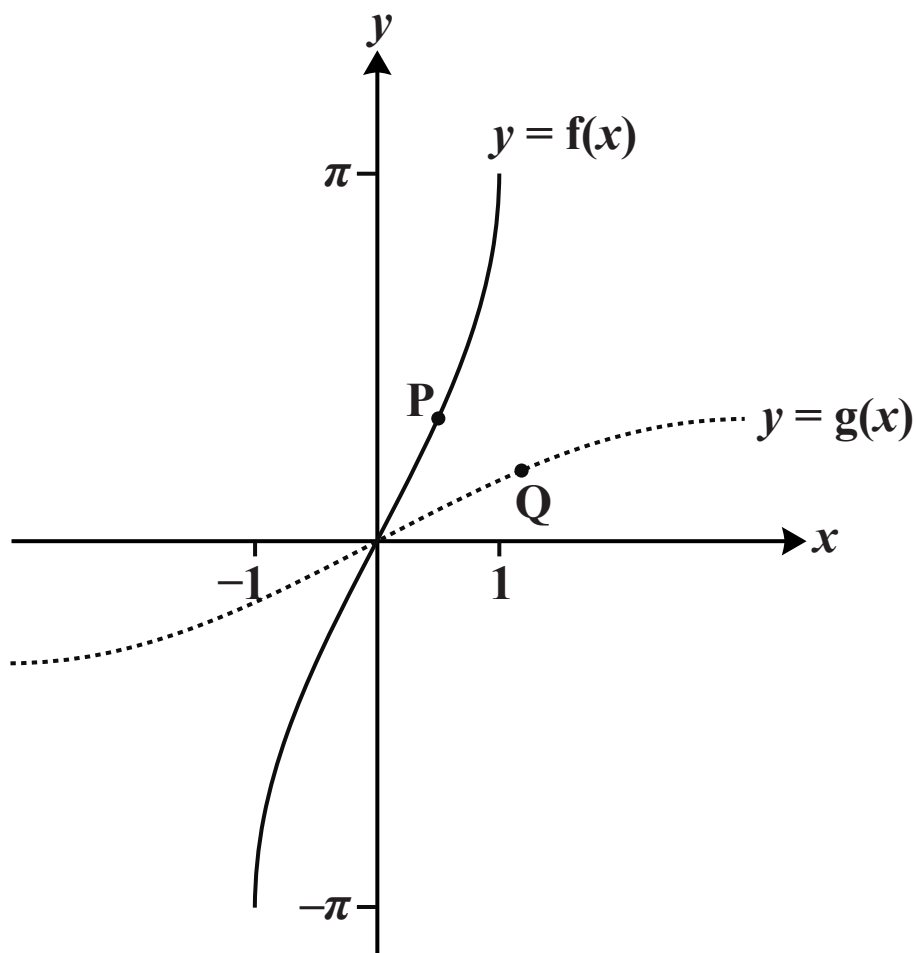


Fig. 6

- (i) Find the y -coordinate of P, giving your answer in terms of π . [2]

The point Q is the reflection of P in $y = x$.

- (ii) Find $g(x)$ and its derivative $g'(x)$. Hence determine the exact gradient of the curve $y = g(x)$ at the point Q.

Write down the exact gradient of $y = f(x)$ at the point P.

- 7 You are given that $f(x)$ and $g(x)$ are odd functions, defined for $x \in \mathbb{R}$.**
- (i) Given that $s(x) = f(x) + g(x)$, prove that $s(x)$ is an odd function. [2]**
- (ii) Given that $p(x) = f(x)g(x)$, determine whether $p(x)$ is odd, even or neither. [2]**

SECTION B (36 marks)

- 8 Fig. 8 below shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x -axis at the point P. The tangent to the curve at P crosses the y -axis at Q.

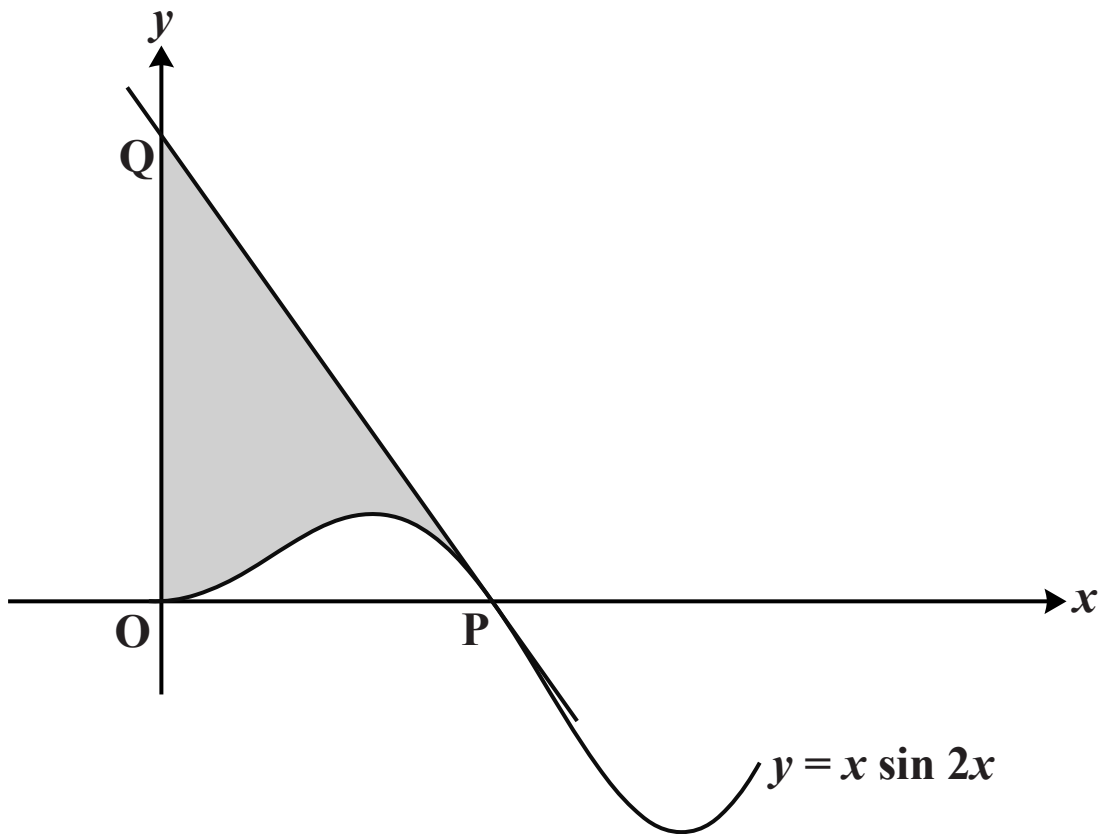


Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x -coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. [4]

(ii) Find, in terms of π , the x -coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. [7]

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. [7]

- 9 Fig. 9 below shows the curve $y = f(x)$, which has a y -intercept at $P(0, 3)$, a minimum point at $Q(1, 2)$, and an asymptote $x = -1$.

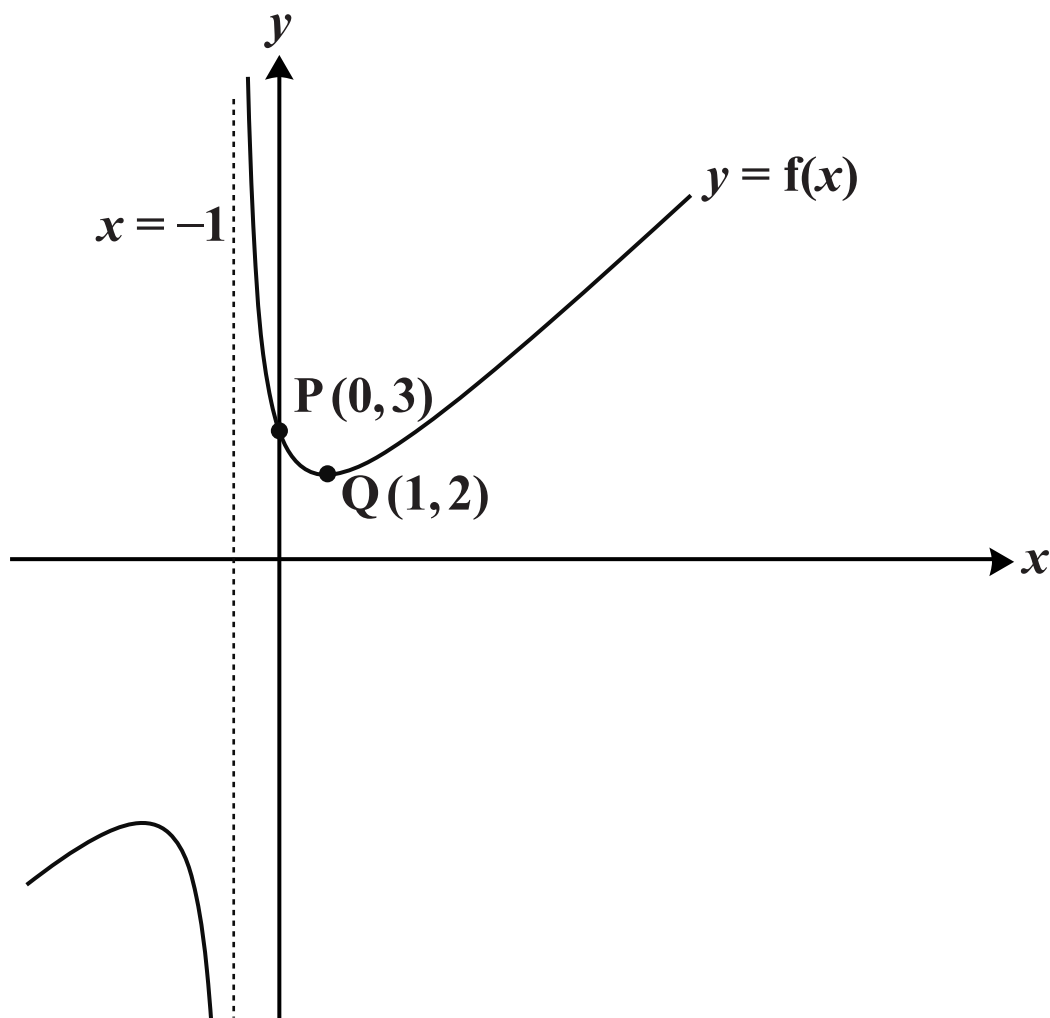


Fig. 9

- (i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to
- (A) $y = 2f(x)$,
- (B) $y = f(x + 1) + 2$. [4]

You are now given that $f(x) = \frac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$. [6]

(iii) Show that $f(x - 1) = x - 2 + \frac{4}{x}$. [3]

(iv) Find $\int_a^b \left(x - 2 + \frac{4}{x}\right) dx$ in terms of a and b .

Hence, by choosing suitable values for a and b , find the exact area enclosed by the curve $y = f(x)$, the x -axis, the y -axis and the line $x = 1$. [5]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.